nuclear fusion

JOURNAL OF PLASMA PHYSICS AND THERMONUCLEAR FUSION

fusion nucléaire

JOURNAL DE PHYSIQUE DES PLASMAS ET FUSION THERMONUCLEAIRE

ядерный синтез

ЖУРНАЛ ПО ФИЗИКЕ ПЛАЗМЫ И ТЕРМОЯДЕРНОМУ СИНТЕЗУ

fusión nuclear

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FIFTH INTERNATIONAL CONFERENCE ON IONIZATION PHENOMENA IN GASES

28 AUGUST — 1 SEPTEMBER, 1961, MUNICH, FEDERAL REPUBLIC OF GERMANY

Sponsored by: Verband Deutscher Physikalischer Gesellschaften, Fachausschuß Plasma- und Gasentladungsphysik (The Association of German Physical Societies, Special Committee for Plasma and Gas Discharge Physics)

For information write to: Sekretariat, Fünfte Internationale Konferenz über Ionisationsphänomene in Gasen, Oskar-von-Miller-Ring 18, Munich I, Federal Republic of Germany

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— fundamental processes in gas discharges (shock processes, radiation, general plasma physics, transport phenomena, surface effects, magnetohydrodynamics);
— discharge types (glow discharge, arc, shock discharge, high-frequency discharge);
— diagnostic methods (spectroscopy, shock waves, microwaves);
— applications.

CONFERENCE ON PLASMA PHYSICS AND CONTROLLED NUCLEAR FUSION RESEARCH

4 — 9 SEPTEMBER, 1961, SALZBURG, AUSTRIA

Sponsored by: International Atomic Energy Agency

For information write to: Conference on Plasma Physics and Controlled Nuclear Fusion
International Atomic Energy Agency
Kaerntnerring 11, Vienna I, Austria

Scope: General theory of plasma and mathematical methods in research motivated by controlled nuclear fusion interest;

Experimental and theoretical problems in
— plasma confinement, stability, oscillations and turbulence;
— plasma compression, heating and acceleration;
— shock waves in plasma;
— interaction of particles and electromagnetic waves with plasma;
— plasma waves and radiation.

CINQUIÈME CONFÉRENCE INTERNATIONALE SUR LES PHÉNOMÈNES D'IONISATION DANS LES GAZ

28 AOÛT — 1er SEPTEMBRE 1961, MUNICH (RÉPUBLIQUE FÉDÉRALE D'ALLEMAGNE)

Organisateur: Verband Deutscher Physikalischer Gesellschaften, Fachausschuß Plasma- und Gasentladungsphysik (Association des sociétés de physique d'Allemagne, Commission spéciale pour la physique des plasmas et des décharges dans les gaz)

Pour tous renseignements, écrire au
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Oskar-von-Miller-Ring 18, Munich I (République fédérale d'Allemagne)

Thème: Recherche pure et appliquée dans les domaines suivantes:
— processus fondamentaux de la décharge dans les gaz (processus de choc, rayonnement, physique générale des plasmas, phénomènes de transport, effets de surface, magnétohydrodynamique),
— types de décharge (décharge luminescente, décharge d'arc, décharge de choc, décharge de haute fréquence),
— méthodes d'investigation (spectroscopie, ondes de choc, micro-ondes),
— applications.

CONFÉRENCE SUR LA PHYSIQUE DES PLASMAS ET LA RECHERCHE CONCERNANT LA FUSION NUCLÉAIRE CONTRÔLÉE

4 — 9 SEPTEMBRE 1961, SALZBOURG (AUTRICHE)

Organisateur: Agence internationale de l'énergie atomique

Pour tous renseignements, écrire à
Conférence sur la physique des plasmas et la recherche
concernant la fusion nucléaire contrôlée
Agence internationale de l'énergie atomique
Kaerntnerring 11, Vienne I (Autriche)

Thème: Théorie générale du plasma et méthodes mathématiques de recherche sur la fusion nucléaire contrôlée;
Problèmes théoriques et expérimentaux que posent
— le confinement, la stabilité, les oscillations et la turbulence du plasma;
— la compression, le chauffage et l'accélération du plasma;
— les ondes de choc dans le plasma;
— l'interaction des ondes électromagnétiques et particules et du plasma;
— les ondes et rayonnements du plasma.

ПЯТАЯ МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ ПО ЯВЛЕНИЯМ ИОНИЗАЦИИ В ГАЗАХ

28 августа — 1 сентября 1961 года, Мюнхен, Федеративная Республика Германия

Организатор: Verband Deutscher Physikalischer Gesellschaften, Fachausschuß Plasma- und Gasentladungsphysik
(Специальный комитет по физике плазменных и газовых разрядов Ассоциации германских физических обществ).

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Тема конференции: Экспериментальные и теоретические проблемы, в частности:
основные процессы в газовых разрядах (процессы столкновения, радиация, общая физика плазмы, явления переноса, поверхностные эффекты, магнитногидродинамика);
типы разрядов (тлеющий разряд, дуговой разряд, ударные разряды, высоко-частотные разряды);
методы измерения (спектроскопия, ударные волны, сантиметровые волны);
практическое применение.

КОНФЕРЕНЦИЯ ПО ИССЛЕДОВАНИЯМ В ОБЛАСТИ ФИЗИКИ ПЛАЗМЫ И УПРАВЛЯЕМОГО ЯДЕРНОГО СИНТЕЗА

4—9 сентября 1961 года, Зальцбург, Австрия

Организатор: Международное агентство по атомной энергии
За информацией просьба обращаться по адресу:
Конференция по исследованиям в области физики плазмы и управляемого ядерного синтеза,
Международное агентство по атомной энергии, Керитнерринг 11, Вена I, Австрия

Тема конференции: Общая теория плазмы и математические методы, применяемые в исследованиях в области управляемого ядерного синтеза.
Экспериментальные и теоретические проблемы:
— удерживание, устойчивость, колебания и турбулентность плазмы;
— сжатие, нагревание и ускорение плазмы;
— воздействие ударной волны на плазму;
— взаимодействие частиц и электромагнитных волн с плазмой;
— волновые явления в плазме и радиации.

QUINTA CONFERENCIA INTERNACIONAL SOBRE FENÓMENOS DE IONIZACIÓN EN LOS GASES

28 de Agosto al 1° de Septiembre de 1961, Munich (República Federal de Alemania)

Patrocinada por la «Verband Deutscher Physikalischer Gesellschaften, Fachausschuß Plasma- und Gasentladungsphysik»
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Temario — Problemas experimentales y teóricos referentes a:
— procesos fundamentales de descargas en los gases (procesos de choque, radiación, física general del plasma, fenómenos de transporte, efectos de superficie, magnetohidrodinámica),
— tipos de descarga (luminiscentes, en arco, de choque, y de alta frecuencia),
— métodos de diagnóstico (espectroscopia, ondas de choque, microondas),
— aplicaciones.

CONFERENCIA SOBRE LAS INVESTIGACIONES EN MATERIA DE FÍSICA DEL PLASMA Y FUSIÓN NUCLEAR CONTROLADA

4 al 9 de Septiembre de 1961, Salzburgo (Austria)

Patrocinada por el Organismo Internacional de Energía Atómica
Para obtener informes, escríbase a: Conferencia sobre las investigaciones en materia de física del plasma y fusión nuclear controlada
Organismo Internacional de Energía Atómica
Kaerntnerring 11, Viena I, Austria

Temario — Teoría general del plasma y métodos matemáticos empleados en las investigaciones motivadas por el interés que suscita la fusión nuclear controlada
Problemas experimentales y teóricos referentes a:
— confinamiento, estabilidad, oscilaciones y turbulencia del plasma,
— compresión, calentamiento y aceleración del plasma,
— ondas de choque en el plasma,
— interacción de partículas y ondas electromagnéticas con el plasma,
— ondas de plasma y radiaciones.

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BEHAVIOUR OF PLASMA IN A ROTATING MAGNETIC FIELD

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An investigation is made of the non-relativistic motion of a charged particle (in a plasma) in an external rotating electromagnetic field of the form: $H_x = H_0 \cos(\omega y/c) \cos \omega t$, $H_y = H_0 \cos(\omega x/c) \sin \omega t$, $H_z =$ constant, $E_x = E_y = 0$, $E_z = H_0 [\sin(\omega x/c) \cos \omega t + \sin(\omega y/c) \sin \omega t]$, with $\omega x/c \ll 1$ and $\omega y/c \ll 1$. If the condition $-1 < (eH_z/mc\omega) < -1 + (eH_0/mc\omega)^2$ is satisfied, then the particle moves away from the Z-axis. The particle energy is $\sim H_0^2 e^2 \overline{r}^2/(mc^2\mathrm{k})$ in °K where $\overline{r}$ is the average distance from the Z-axis.

The motion of the plasma is then investigated taking into account its proper electromagnetic field. The following transport equation is used: $nm\,\dot{\mathbf{v}} + \mathbf{v}\nabla\cdot\mathbf{v} = nq\,(\mathbf{E} + \mathbf{v}\times\mathbf{H}/c) - \nabla\psi - nm\nabla\varphi + \mathbf{p}$. The assumptions are: a plasma consisting of equal populations of electrons and deuterons with density $\sim 10^{15}/\mathrm{cm}^3$, $\omega \leq 10^{10}/\mathrm{sec}$, $H_0 \sim 10^3$ G, $v \leq 0.1\,c$. At $t = 0$ it is assumed that $T = 10^6$ °K and $v = 0$ and that the derivatives of the plasma density with respect to the space variables are negligible compared to other terms of the transport equation. An expansion in powers of v/c is used. Zeroth, first and second order approximations are calculated using the Laplace transformation. Up to the second order of approximation, the field causes neither a durable change in plasma density nor a charge separation.

Oscillations of four different frequencies appear in the plasma. At a definite frequency of the rotating field there appears a resonance phenomenon in which the amplitude of oscillation increases linearly with t. At resonance the mean energy per ion (in °K) transferred directly to the ionic part of the plasma increases with time as follows: $(1/192\,\pi^2)\,(e^2 m/c^4 M^4 \mathrm{k})\,(H_0^4 H_z^2/n_0^2)\,t^2$. This means, for example, that in an axial field of 10^4G and a rotating field of amplitude 10^3G, the time necessary to provide energy corresponding to 10^8 °K (disregarding losses) is ~ 0.3 sec.

1. Introduction

To obtain controlled fusion it is necessary to have very hot plasma; besides, the plasma must be confined for sufficiently long time within a limited space. Practically, the only way to satisfy these requirements is to provide adequate magnetic fields. Therefore it is important to know the behaviour of plasma in particular configurations of magnetic fields.

One possible configuration might be a rotating magnetic field in the presence of a constant homogeneous magnetic field perpendicular to the former. A preliminary investigation of the behaviour of plasma in this configuration of fields, under certain simplifying assumptions, is undertaken in the present paper.

2. External magnetic field

We assume that the external magnetic field has the following form

$$\left.\begin{aligned} H_x &= H_0 \cos(\omega y/c) \cos \omega t \\ H_y &= H_0 \cos(\omega x/c) \sin \omega t \\ H_z &= \text{constant} \\ E_x &= E_y = 0 \\ E_z &= H_0 [\sin(\omega x/c) \cos \omega t + \sin(\omega y/c) \sin \omega t]. \end{aligned}\right\} \qquad (1)$$

Near the Z-axis this field corresponds to the configuration of the rotating magnetic field:

$$\left.\begin{aligned} H_x &= H_0 \cos \omega t \\ H_y &= H_0 \sin \omega t \\ H_z &= \text{constant} \\ E_x &= E_y = 0 \\ E_z &= (H_0 \omega/c)\,(x \cos \omega t + y \sin \omega t), \end{aligned}\right\} \qquad (2)$$

provided the following conditions are satisfied:

$$\omega x/c \ll 1, \quad \omega y/c \ll 1. \qquad (3)$$

3. The motion of a charged particle in the rotating field

As an initial problem let us consider the motion of a charged particle in the field defined by Eq. (2). Starting from the non-relativistic equation of motion:

$$m\ddot{\mathbf{r}} = e\,\mathbf{E} + (e/c)\,\dot{\mathbf{r}} \times \mathbf{H}, \qquad (4)$$

we obtain, after taking into consideration Eq. (2), the equations for the separate components:

$$\begin{aligned} \frac{\mathrm{d}^2 x}{\mathrm{d}\tau^2} &= \Omega_3 \frac{\mathrm{d}y}{\mathrm{d}\tau} - (\Omega_0 \sin\tau)\,\frac{\mathrm{d}z}{\mathrm{d}\tau} \\ \frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} &= -\,\Omega_3 \frac{\mathrm{d}x}{\mathrm{d}\tau} + (\Omega_0 \cos\tau)\,\frac{\mathrm{d}z}{\mathrm{d}\tau} \\ \frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} &= \Omega_0 \frac{\mathrm{d}}{\mathrm{d}\tau}\,(x \sin\tau - y\cos\tau), \end{aligned} \qquad (5)$$

where

$$\tau = \omega t, \quad \Omega_0 = e\,H_0/(mc\omega), \quad \Omega_3 = e\,H_z/(mc\omega).$$

Integrating the third equation of Eq. (5) we get

$$\frac{dz}{d\tau} = \Omega_0 (x \sin \tau - y \cos \tau) + C_3 . \qquad (6)$$

Substituting Eq. (6) into the first two equations of Eq. (5), we obtain a system of equations that can be written thus

$$\frac{d^2 U}{d\tau^2} = -i\Omega_3 \frac{dU}{d\tau} - \frac{1}{2}\Omega_0^2 U + \frac{1}{2}\Omega_0^2 U^* e^{2i\tau} + i\Omega_0 C_3 e^{i\tau}, \qquad (7)$$

where $U = x + i\, y$.

Substituting then

$$U = f\, e^{i\tau}, \qquad (8)$$

we obtain an equation with constant coefficients

$$\frac{d^2 f}{d\tau^2} + i(2+\Omega_3)\frac{df}{d\tau} + \left(\frac{1}{2}\Omega_0^2 - \Omega_3 - 1\right) f - \frac{1}{2}\Omega_0^2 f^* = i\,\Omega_0 C_3. \qquad (9)$$

Solving Eq. (9) and substituting the result into Eq. (8), we obtain

$$U = \left[u_1 + i\left(u_2 + \frac{\Omega_0 C_3}{\Omega_0^2 - \Omega_3 - 1}\right)\right] e^{i\tau}, \qquad (10)$$

where

$$u_1 = -(2+\Omega_3)\left[\frac{\Omega_1 C_1}{\Omega_1^2 + \Omega_3 + 1}\cos(\Omega_1 \tau + \varphi_1) + \frac{\Omega_2 C_2}{\Omega_2^2 + \Omega_3 + 1}\cos(\Omega_2 \tau + \varphi_2)\right]$$

$$u_2 = C_1 \sin(\Omega_1 \tau + \varphi_1) + C_2 \sin(\Omega_2 \tau + \varphi_2)$$

$$\Omega_{1,2} = \sqrt{1/2}\,[2 + \Omega_0^2 + \Omega_3^2 + 2\Omega_3 \pm \sqrt{\Delta}]^{\frac{1}{2}}$$

$$\Delta = (2 + \Omega_0^2 + \Omega_3^2 + 2\Omega_3)^2 - 4(\Omega_3 + 1)(\Omega_3 + 1 - \Omega_0^2).$$

C_1, C_2, C_3, φ_1, φ_2 are constants dependent on the initial state of the particle.

The condition for the particle to move within a limited distance from the Z-axis is that $\Omega_{1,2}$ must be real. This leads to the inequality

$$(\Omega_0^2 + \Omega_3^2 + 2\Omega_3 + 2)^2 > 4(\Omega_3 + 1)(\Omega_3 + 1 - \Omega_0^2) > 0 . \qquad (11)$$

The left side of the inequality is always satisfied. The right side leads to a condition which can be written as

either

$$e H_z/(m c\omega) < -1$$

or

$$e H_z/(m c \omega) > -1 + [e H_0/(m c \omega)]^2 . \qquad (12)$$

If there are two kinds of particles, negative electrons and positive ions, by a proper choice of H_z, H_0 and ω we can obtain the four possible cases, viz:

(a) both kinds of particles move within a limited distance from the Z-axis;

(b) both kinds of particles move away from the Z-axis;

(c, d) the particles of one kind move away and those of the other kind remain confined.

The energy achieved by a particle can be estimated on the basis of Eq. (6). Assuming that the average distance from the Z-axis is $\bar{r}$, the average energy of the particle will be of the order

$$T_K = \frac{H_0^2 e^2 \bar{r}^2}{m c^2 k} (°K) , \qquad (13)$$

where k denotes the Boltzmann constant. If $H_0 = 1000$ G and $\bar{r} = 10$ cm, the energy of the deuterons ($M = 3{,}3 \cdot 10^{-24}$g) will be of the order

$$T_K \sim 5.10^7 (°K) .$$

4. The motion of plasma taking into account its proper electromagnetic field

4.1. Initial equations and assumptions

Let us start from the transport equation for charged particles, page 97 of [1],

$$n m \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\nabla\cdot\mathbf{v}\right) = n q \left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{H}}{c}\right) - \nabla\psi - n m \nabla\varphi + \mathbf{p} . \qquad (14)$$

We assume that:

1) The electron density n and the ion density N are of the order of 10^{15} cm^{-3}.
2) The rotating field frequency $\omega \leq 10^{10}$ sec^{-1}.
3) The amplitude of the external magnetic field is of the order of 10^3 G.
4) Plasma temperature at $t = 0$ is of the order of 10^6 °K; this temperature has been achieved by other means.
5) Particle velocities are at least one order of magnitude lower than the velocity of light.
6) Deuterons serve as ions.

Let us introduce new variables

$$\left.\begin{aligned} &\mathbf{w} = \mathbf{v}/c \quad \text{(electrons)} \\ &\mathbf{W} = \mathbf{V}/c \quad \text{(ions)} \\ &\omega_p = \sqrt{4\pi e^2 n_0/\mu}, \quad \mu = m M/(m+M) \\ &x, y, z \rightarrow x_1 x_2 x_3 \\ &\xi_k = (\omega/c)\, x_k, \quad \tau = \omega_p t . \end{aligned}\right\} \qquad (15)$$

n_0 is the plasma density at $t = 0$.

Accepting the above assumptions, we note that the first terms of the left and right side of Eq. (14) are the most important because the other terms are smaller by at least three orders of magnitude. In the notation of Eq. (15) the transport equations for the electrons and ions will be, respectively:

$$\left.\begin{aligned} \frac{\partial \mathbf{w}}{\partial \tau} &= -\frac{e}{m\,\omega_p c}(\mathbf{E} + \mathbf{w}\times\mathbf{H}) \\ \frac{\partial \mathbf{W}}{\partial \tau} &= +\frac{e}{M\,\omega_p c}(\mathbf{E} + \mathbf{W}\times\mathbf{H}) . \end{aligned}\right\} \qquad (16)$$

To the above equations the equation of continuity and the Maxwell equations should be added. In the new variables of Eq. (15) the equations are

$$
\left.\begin{aligned}
&\frac{\partial n}{\partial \tau}+\frac{\omega}{\omega_p}\nabla\cdot(n\,\mathbf{w})=0\\
&\frac{\partial N}{\partial \tau}+\frac{\omega}{\omega_p}\nabla\cdot(N\,\mathbf{W})=0\\
&\qquad \varrho=e\,(N-n),\ \mathbf{j}=e\,c\,(N\mathbf{W}-n\,\mathbf{w})\\
&\nabla\times\mathbf{E}=-\frac{\omega_p}{\omega}\frac{\partial\mathbf{H}}{\partial\tau}\\
&\nabla\times\mathbf{H}=-\frac{\omega_p}{\omega}\mathbf{E}+\frac{4\pi}{\omega}\mathbf{j}\\
&\nabla\cdot\mathbf{E}=\frac{4\pi c}{\omega}\varrho\\
&\nabla\cdot\mathbf{H}=0\,.
\end{aligned}\right\}\quad(17)
$$

The system, Eqs. (16) and (17) is nonlinear. To solve we expand all variables in powers of a small parameter δ

$$
\left.\begin{aligned}
&\mathbf{w}=\sum_l\mathbf{w}_l\,\delta^l\,, && \mathbf{W}=\sum_l\mathbf{W}_l\,\delta^l\\
&\mathbf{E}=\sum_l\mathbf{E}_l\,\delta^l\,, && \mathbf{H}=\sum_l\mathbf{H}_l\,\delta^l\\
&n=\sum_l n_l\,\delta^l\,, && N=\sum_l N_l\,\delta^l\\
&\varrho=\sum_l\varrho_l\,\delta^l\,, && \mathbf{j}=\sum_l\mathbf{j}_l\,\delta^l\,.
\end{aligned}\right\}\quad(18)
$$

To determine δ we assume that $\mathbf{w}_0=\mathbf{W}_0=0$ and $\mathbf{w}_1$ is of the order of unity — then the parameter δ will be of the order of v/c for electrons.

4.2. The initial conditions

We assume that at $t=0$ the plasma is at rest, i.e.

$$\mathbf{w}\,(0)=\mathbf{W}\,(0)=\mathbf{j}\,(0)=0\,,$$

and is neutral

$$n\,(0)=N(0),\ \ \varrho\,(0)=0\,.$$

We assume also that at $t=0$ the plasma is practically homogeneous; this means that the derivatives of plasma density with respect to spatial variables, are negligible in comparison with the other terms of the equations.

4.3. Zero-order approximation

Substituting the expansion, Eq. (18), into Eqs. (16), (17), and comparing the coefficients for the zero power of δ we obtain a zero-order approximation:

$$
\left.\begin{aligned}
&\frac{\partial\mathbf{w}_0}{\partial\tau}=-\frac{e}{m\,c\,\omega_p}(\mathbf{E}_0+\mathbf{w}_0\times\mathbf{H}_0)\\
&\frac{\partial\mathbf{W}_0}{\partial\tau}=\frac{e}{M\,c\,\omega_p}(\mathbf{E}_0+\mathbf{W}_0\times\mathbf{H}_0)\\
&\frac{\partial n_0}{\partial\tau}+\frac{\omega}{\omega_p}\nabla\,(n_0\,\mathbf{w}_0)=0,\ \varrho_0=e\,(N_0-n_0)\\
&\frac{\partial N_0}{\partial\tau}+\frac{\omega}{\omega_p}\nabla(N_0\mathbf{W}_0)=0,\ \mathbf{j}_0=e\,c\,(N_0\mathbf{W}_0-n_0\mathbf{w}_0),
\end{aligned}\right\}\quad(19)
$$

and the Maxwell equation for the quantities with the index $l=0$. From the assumption $\mathbf{w}_0=\mathbf{W}_0=0$ it follows that $\mathbf{E}_0=0$ and $\mathbf{H}_0$ does not depend on time. We are going to assume that the homogeneous field along the X_3-(or Z-) axis is of zero order and is an order of magnitude larger than the amplitude of the rotating field. Thus we obtain in the zero-order approximation the following solution:

$$
\begin{aligned}
&\mathbf{E}_0=0\quad \mathbf{w}_0=0\quad \mathbf{W}_0=0\quad \mathbf{j}_0=0\quad \varrho_0=0\\
&\mathbf{H}_0\,(0{,}0{,}H_3),\ \ H_3=\text{constant},\ \ n_0=N_0=\text{constant},
\end{aligned}\quad(20)
$$

where H_3 is x_3- or z-component of the external magnetic field, see Eq. (2).

4.4. First-order approximation

Proceeding in the same way as in Section 4.3, and taking into account Eq. (20) we obtain the equations of the first-order approximation

$$
\left.\begin{aligned}
&\frac{\partial\mathbf{w}_1}{\partial\tau}=-\frac{e}{m\,c\,\omega_p}(\mathbf{E}_1+\mathbf{w}_1\times\mathbf{H}_0)\\
&\frac{\partial\mathbf{W}_1}{\partial\tau}=\frac{e}{M\,c\,\omega_p}(\mathbf{E}_1+\mathbf{W}_1\times\mathbf{H}_0)\\
&\frac{\partial n_1}{\partial\tau}+\frac{\omega}{\omega_p}\nabla\cdot(n_0\,\mathbf{w}_1)=0\quad \varrho_1=e\,(N_1-n_1)\\
&\frac{\partial N_1}{\partial\tau}+\frac{\omega}{\omega_p}\nabla\cdot(N_0\mathbf{W}_1)=0\quad \mathbf{j}_1=e\,c\,n_0\,(\mathbf{W}_1-\mathbf{w}_1)\,,
\end{aligned}\right\}\quad(21)
$$

and the Maxwell equations for the quantities with index $l=1$.

Consider the electromagnetic field $\mathbf{E}$ and $\mathbf{H}$ to be a superposition of the external field $\mathbf{E}_0$, $\mathbf{H}_0$ expressed by Eq. (1) and of the proper plasma field $\boldsymbol{\varepsilon}$, $\mathbf{h}$,

$$\mathbf{E}=\boldsymbol{\varepsilon}+\mathbf{E}_0\,,\quad \mathbf{H}=\mathbf{h}+\mathbf{H}_0\,.\qquad(22)$$

Assuming, further, that the proper plasma field at $t=0$ is equal to zero, we can obtain the solution of the system of equations, Eq. (21), without any difficulty after appropriate calculations using the Laplace transformation:

$$
\left.\begin{aligned}
&E_{11}=E_{12}=0,\,H_{13}=0,\,W_{11}=W_{12}=w_{11}=w_{12}=0\\
&\quad j_{11}=j_{12}=0,\quad \varrho_1=0,\quad n_1=N_1=0\\
&E_{13}=\frac{H_0}{\delta}\left(\sin\xi_1\cos\Omega_1\tau+\frac{\Omega}{\Omega_1}\sin\xi_2\sin\Omega_1\tau\right)\\
&H_{11}=\frac{H_0}{\delta}\left(\frac{\Omega^2}{\Omega_1{}^2}\cos\xi_2\cos\Omega_1\tau+\frac{1}{\Omega_1{}^2}\cos\xi_2\right)\\
&H_{12}=\frac{H_0}{\delta}\left(\frac{\Omega}{\Omega_1}\cos\xi_1\sin\Omega_1\tau\right)\\
&w_{13}=-\frac{e\,H_0}{\delta\,m\,c\,\omega_p\,\Omega_1}\times\\
&\qquad\left[\sin\xi_1\sin\Omega_1\tau-\frac{\Omega}{\Omega_1}\sin\xi_2\,(\cos\Omega_1\tau-1)\right]\\
&W_{13}=\frac{e\,H_0}{\delta\,m\,c\,\omega_p\,\Omega_1}\times\\
&\qquad\left[\sin\xi_1\sin\Omega_1\tau-\frac{\Omega}{\Omega_1}\sin\xi_2\,(\cos\Omega_1\tau-1)\right]\\
&\quad\Omega=\frac{\omega}{\omega_p}\,,\qquad \Omega_1=\sqrt{1+\Omega^2}\,.
\end{aligned}\right\}\quad(23)
$$

4.5. Second-order approximation

In the second-order approximation, having taken account of the solutions of the zero and first order, we obtain the equations

$$\left.\begin{aligned}
&\frac{\partial \mathbf{w}_2}{\partial \tau}+\frac{e}{m c \omega_p}\mathbf{w}_2\times\mathbf{H}_0=-\frac{e}{m c \omega_p}(\mathbf{E}_2+\mathbf{b})\\
&\frac{\partial \mathbf{W}_2}{\partial \tau}-\frac{e}{M c \omega_p}\mathbf{W}_2\times\mathbf{H}_0=\frac{e}{M c \omega_p}(\mathbf{E}_2+\mathbf{B})\\
&\mathbf{b}=\mathbf{w}_1\times\mathbf{H}_1,\quad \mathbf{B}=\mathbf{W}_1\times\mathbf{H}_1\\
&\frac{\partial n_2}{\partial \tau}+\Omega\nabla\cdot(n_0\mathbf{w}_2)=0,\quad \varrho_2=e(N_2-n_2)\\
&\frac{\partial N}{\partial \tau}+\Omega\nabla\cdot(N_0\mathbf{W}_2)=0,\quad \mathbf{j}_2=e c n_0(\mathbf{W}_2-\mathbf{w}_2),
\end{aligned}\right\}\tag{24}$$

and the Maxwell equations for quantities with the index $l=2$. In this approximation

$$w_{23}=W_{23}=0,\quad \varepsilon_{23}=0,\quad j_{23}=0,\tag{25}$$

which results from the fact that

$$w_{12}=w_{11}=W_{12}=W_{11}=0,\quad b_3=B_3=0.$$

Applying the Laplace transformation to Eq. (24) and introducing complex variables:

$$\left.\begin{aligned}
&w=w_{21}+\mathrm{i}\,w_{22}\quad W=W_{21}+\mathrm{i}\,W_{22}\\
&\eta=\varepsilon_{21}+\mathrm{i}\,\varepsilon_{22}\quad b=b_1+\mathrm{i}\,b_2\quad B=B_1+\mathrm{i}\,B_2\\
&j=j_{21}+\mathrm{i}\,j_{22}\quad H=H_{11}+\mathrm{i}\,H_{12}\\
&\frac{e H_3}{m c\omega_p}=\Omega_e\qquad \frac{e H_3}{M c\omega_p}=\Omega_i,
\end{aligned}\right\}\tag{26}$$

we obtain the equations

$$\left.\begin{aligned}
&\hat{w}=-\frac{e}{m c\omega_p}\,\frac{1}{p-\mathrm{i}\Omega_e}(\hat{\eta}+\hat{b})\\
&\hat{W}=\frac{e}{M c\omega_p}\,\frac{1}{p+\mathrm{i}\Omega_i}(\hat{\eta}+\hat{B})\\
&\Omega^2\Big(\frac{1}{2}\sum_l\hat{\eta}_{ll}-\frac{1}{2}\hat{\eta}_{11}{}^*+\frac{1}{2}\hat{\eta}_{22}{}^*-\mathrm{i}\,\hat{\eta}_{12}{}^*\Big)\\
&\qquad\qquad=p^2\hat{\eta}+\frac{4\pi}{\omega_p}p\,\hat{j},
\end{aligned}\right\}\tag{27}$$

where p is a transformation parameter, the asterisk denotes a conjugate variable, $\hat{w}$ is the transform of w, etc., and the subscript symbol l denotes $\partial/\partial\xi_l$.

From the Eq. (27) we can obtain the equation for $\hat{\eta}$ which can be written in the form

$$\left.\begin{aligned}
&\Omega^2\Big(\frac{1}{2}\sum_l\hat{\eta}_{ll}-\frac{1}{2}\hat{\eta}^*_{11}+\frac{1}{2}\hat{\eta}^*_{22}-\mathrm{i}\,\hat{\eta}^*_{12}\Big)-A\hat{\eta}=S\hat{G},\\
&\text{where}\\
&S=\frac{m-M}{m M}\,p\,\frac{p+\mathrm{i}\Omega_{ie}}{(p+\mathrm{i}\Omega_i)(p-\mathrm{i}\Omega_e)}\\
&\Omega_{ie}=\frac{e H_3}{(M-m)\,\omega_p c}\\
&A=p^2\frac{(p+\mathrm{i}\Omega_i)(p-\mathrm{i}\Omega_e)+1}{(p-\mathrm{i}\Omega_i)(p-\mathrm{i}\Omega_e)}\\
&G=\mathrm{i}DH\\
&D=\frac{e H_0}{\delta c\omega_p\Omega_1}\times\\
&\qquad\Big[\sin\xi_1\sin\Omega_1\tau-\frac{\Omega}{\Omega_1}\sin\xi_2(\cos\Omega_1\tau-1)\Big].
\end{aligned}\right\}\tag{28}$$

Retaining in the expressions for D and H only the largest terms (at least two orders of magnitude larger than the remaining ones), we obtain

$$\hat{G}=\mathrm{i}F[\sin(\xi_1+\xi_2)+\sin(\xi_1-\xi_2)]\frac{1}{p^2+\Omega_1{}^2}\tag{29}$$

$$F=\frac{e H_0{}^2}{2\delta^2 c\,\omega_p\,\Omega_1{}^2}.$$

Assuming $\hat{\eta}$ in the form

$$\hat{\eta}=\hat{F}_1\sin(\xi_1+\xi_2)+\hat{F}_2\sin(\xi_1-\xi_2),$$

and substituting it into Eq. (28), we obtain, after proper calculations,

$$\left.\begin{aligned}
&\hat{F}_1=F\frac{M-m}{m M}\,\frac{1}{Q}\,\frac{1}{p^2+\Omega_1{}^2}(Q_1+\varphi)\\
&\hat{F}_2=F\frac{M-m}{m M}\,\frac{1}{Q}\,\frac{1}{p^2+\Omega_1{}^2}(Q_2+\varphi),
\end{aligned}\right\}\tag{30}$$

where

$$\left.\begin{aligned}
Q=\;&2\,\Omega^2\big[(p^2+\Omega_i^2)(p^2+\Omega_e^2)+p^2+\Omega_i\Omega_e\big]\\
&+p^2\big[(p^2+\Omega_i^2)(p^2+\Omega_e^2)+2(p^2+\Omega_i\Omega_e)+1\big]\\
Q_1=\;&\frac{\Omega^2(1+\mathrm{i})}{p}[p^3+(\Omega_i-\Omega_e-\Omega_{ie})p^2\\
&+(\Omega_i\Omega_e+\Omega_{ie}\Omega_i-\Omega_{ie}\Omega_e)p-\Omega_i\Omega_e\Omega_{ie}]\\
Q_2=\;&\frac{\Omega^2(\mathrm{i}-1)}{p}[p^3+(\Omega_e-\Omega_i+\Omega_{ie})p^2\\
&+(\Omega_i\Omega_e+\Omega_{ie}\Omega_i-\Omega_{ie}\Omega_e)p+\Omega_i\Omega_e\Omega_{ie}]\\
\varphi=\;&p\big[\{-(\Omega_{ie}+\Omega_e-\Omega_i)p^2+p-\Omega_{ie}\Omega_i\Omega_e\}\\
&+\mathrm{i}\{p^3+(\Omega_i\Omega_e+\Omega_i\Omega_{ie}-\Omega_e\Omega_{ie})p+\Omega_{ie}\}\big].
\end{aligned}\right.\tag{31}$$

Now we substitute the expressions for $\hat{F}_1$ and $\hat{F}_2$ into Eq. (30) and obtain the Laplace transform of the electric field $\hat{\eta}$ and the transform of electron and ion velocities of the plasma:

$$\left.\begin{aligned}
&\hat{w}=-\frac{e}{m c\omega_p}\frac{p+\mathrm{i}\Omega_e}{p^2+\Omega_e{}^2}[\alpha_1\sin(\xi_1+\xi_2)+\alpha_2\sin(\xi_1-\xi_2)]\\
&\hat{W}=\frac{e}{M c\omega_p}\frac{p-\mathrm{i}\Omega_i}{p^2+\Omega_i{}^2}[\beta_1\sin(\xi_1+\xi_2)+\beta_2\sin(\xi_1-\xi_2)]\\
&\alpha_1=\hat{F}_1-\mathrm{i}\frac{F}{m}\frac{1}{p^2+\Omega_1{}^2},\quad \alpha_2=\hat{F}_2-\mathrm{i}\frac{F}{m}\frac{1}{p^2+\Omega_1{}^2}\\
&\beta_1=\hat{F}_1+\mathrm{i}\frac{F}{M}\frac{1}{p^2+\Omega_1{}^2},\quad \beta_2=\hat{F}_2+\mathrm{i}\frac{F}{M}\frac{1}{p^2+\Omega_1{}^2}.
\end{aligned}\right\}\tag{32}$$

After the application of the inverse transformation we obtain the solution of the second-order approximation. However, as such expressions would be too complicated and awkward for discussion we shall discuss the transform instead, Eqs. (30)—(32), and apply the inverse transformation only in such cases where it is necessary to obtain numerical results.

5. Discussion

5.1. The problem of the change of plasma density

From the expressions Eq. (31) and (32) it follows that in the velocities of both plasma components the periodical terms as well as the non-periodical

are present, the latter being independent of time. From the point of view of a durable change in plasma density the latter are the most important. Introducing $p=0$ into Eq. (31) and (32) we find that the time-independent terms of the plasma velocities satisfy the equations

$$\nabla \cdot \mathbf{w}_2 = 0, \quad \nabla \cdot \mathbf{W}_2 = 0 . \tag{33}$$

This means, that up to the second order of approximation, the field concerned causes neither a durable change in plasma density nor a charge separation.

5.2. Frequencies of plasma oscillation

The frequencies plasma oscillations are given by the singular points of the expressions, Eq. (32). For electrons these will be the points

$$\begin{aligned} p &= \pm \mathrm{i}\, \Omega_e \\ p &= \pm \mathrm{i}\, \Omega_1 . \end{aligned} \tag{34}$$

and the singular points of $\hat{F}_1$ and $\hat{F}_2$.

For ions the points will be

$$\begin{aligned} p &= \pm \mathrm{i}\, \Omega_i \\ p &= \pm \mathrm{i} \Omega_1 , \end{aligned} \tag{35}$$

and the singular points of $\hat{F}_1$ and $\hat{F}_2$.

The latter are found by the assumption $Q=0$ in Eq. (31). This expression is of the third degree in p^2; therefore, zero points of this polynomial can be found without difficulty. We shall not perform this computation because it is complicated and not interesting; we shall confine ourselves to the statement that, on the basis of the form of the expression for Q, it can be shown that all the three roots are imaginary and that each root is different. This means that in the plasma there appear oscillations of three additional different frequencies

$$\begin{aligned} p &= \pm \mathrm{i}\, \Omega_2 \\ p &= \pm \mathrm{i}\, \Omega_3 \\ p &= \pm \mathrm{i}\, \Omega_4 . \end{aligned} \tag{36}$$

5.3. Resonance phenomena

There arises an interesting question whether it is possible to find a frequency of the rotating field such as to cause a resonance of the ion part of the plasma.

Putting $p^2 = -\Omega_i^2$ in the expression for Q in Eq. (31), we obtain for the resonance frequency of the rotating field the formula

$$\Omega_r^2 = \frac{\Omega_i \left[1 + 2\, \Omega_i (\Omega_e - \Omega_i)\right]}{2\, (\Omega_e - \Omega_i)} . \tag{37}$$

With the accepted assumptions this frequency in ordinary units is approximately equal to

$$\omega_r \simeq \sqrt{\frac{2\pi\, n_0\, e^2}{M}} . \tag{38}$$

It follows from the above formulae that the resonance frequency of the rotating field is not equal to the cyclotron-frequency of the ions but has another value, practically independent of H_3 ($=H_z$). This value is of the order of the proper frequency of the ion part of the plasma.

Also from Eq. (37) it follows that, for plasma in a dispersed state when the condition

$$2\, \Omega_e\, \Omega_i \gg 1 \tag{39}$$

is satisfied, the resonance frequency of the rotating field is approximately equal to the ion cyclotron frequency,

$$\Omega_r \simeq \Omega_i .$$

Thus, in this case we have an ordinary cyclotron resonance phenomenon.

5.4. The problem of energy exchange between the electromagnetic field and the plasma

Power density transferred to plasma by the electromagnetic field is expressed by $\mathbf{E} \cdot \mathbf{j}$. Taking into consideration the power density transferred to the ion part of the plasma and taking into account only the resonance terms we get from Eqs. (30), (31), (32), an expression for average energy transferred per single ion, in kinetic temperature units:

$$T = \frac{1}{192\, \pi^2}\, \frac{e^2\, m}{c^4\, M^4\, \mathrm{k}}\, \frac{H_0^4 H_3^2}{n_0^2} \left(t^2 + \frac{t}{\omega_i}\right) (^\circ\,\mathrm{K}) . \tag{40}$$

If the axial field $H_z = H_3 = 10^4$ G and the amplitude of the rotating magnetic field $H_0 = 10^3$ G the time necessary to provide energy corresponding to 10^8 °K, disregarding losses, amounts to about 0,3 sec.

6. Final remarks

All the above results have of course an approximate character because the assumptions take into account only the volume effects. In practical equipment plasma is never homogeneous. In addition, the assumed initial conditions disregard the way of switching in the rotating field.

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AXIAL CONDUCTION AND RADIATION LOSSES IN A STABILISED LINEAR PINCH

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This paper presents a theory of conduction and radiation losses in a linear pinched discharge under steady conditions. The model selected is one in which the ohmic heating in a thin skin of current is equated to the radiation and axial conduction losses, the discharge being considered at a uniform pressure determined by the Bennett relation modified to include the effect of a possible completely trapped axial magnetic field. Formulae for temperature and other physical quantities are presented and limiting forms considered in a variety of circumstances. The effect of thermoelectric phenomena is considered and conditions under which the treatment is likely to be applicable discussed.

1. Introduction

This paper investigates theoretically the significance of thermal conduction loss at the electrodes of a stabilised linear pinched discharge. BRAGINSKII and SHAFRANOV [1], and HAINES [2] have also considered this problem but these authors have investigated models differing considerably from that discussed here.

The present work has been developed from that of KAUFMAN and FURTH [3]. They consider a quiescent plasma in which axial conduction loss through the electrodes is maintained by joule heating. Adopting a one-dimensional formulation and assuming the electrodes to be held steady at some known temperature, they derive a relation connecting voltage across the discharge and maximum temperature generated. They also obtain the temperature distribution as a function of distance from the centre of the discharge.

Under suitable conditions, the axial current in a discharge is at first confined by inductive effects to a thin layer at the surface of the discharge. According to TAYLER [4], when the current has penetrated to an effective depth of about 15% of the radius of the discharge, instabilities cause break-up and the gas is no longer adequately confined. Within the main body of the discharge the pressure should be approximately uniform since in this region there is no appreciable current. These considerations suggest a simple model which is susceptible to mathematical treatment and based on the following assumptions:

(1) The gas of the discharge is fully ionised throughout a cylinder, but the axial current only passes through a skin at the surface occupying a fraction λ of the cross section of the cylinder. This assumption replaces the consideration of inductive effects. The general problem of how the axial current is distributed in a cylindrical conductor has been considered by HAINES [2].

(2) The current in the skin is determined by the usual resistivity formula.

(3) The pressure is uniform throughout the discharge.

(4) Any longitudinal magnetic field is considered to be completely trapped. This assumption is justified to some extent by the work of ALLIBONE *et al.* [5].

(5) The discontinuity in the axial magnetic field, at the surface of the cylinder, gives rise to an azimuthal sheet current which we assume is uniform within a skin. This skin occupies a fraction μ of the cross section of the discharge.

(6) The only loss processes operative are axial thermal conduction and radiation losses through the emission of bremsstrahlung.

(7) Sheath effects close to the electrodes are neglected.

(8) The temperature of the plasma is uniform across any cross section of the discharge perpendicular to the axis but varies along the axis of the tube.

(9) The ion and electron temperatures are equal.

(10) The discharge has reached a steady state.

(11) Temperature and current are related by the pinch relation of BENNETT [6], modified to include the effect of an applied longitudinal magnetic field.

The assumptions (1) and (5) appear to contradict assumption (10) as well as the concept of appreciable heat loss, since the diffusion of a magnetic field into a conducting fluid proceeds at a rate proportional to that of the diffusion of heat. Thus it is necessary to show that there exists an interesting range of temperature T, pinch radius a and length L of the discharge in which

$$\frac{\delta_e}{\delta_n} \ll \frac{a}{L}, \tag{1}$$

where δ_e is the electrical skin-depth and δ_n is the heat skin-depth. Generally

$$\frac{\delta_e}{\delta_n} = \frac{1}{2\pi}\sqrt{\frac{\eta}{\varkappa}}, \tag{2}$$

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where η is the electrical resistivity and $\varkappa = K/(\varrho C_g)$ is the thermal diffusivity, ϱ and C_g being the density and specific heat per gram of plasma, respectively. Anticipating the formulae of Eq. (4) and the physical constant of Eq. (32) one obtains

$$\frac{\delta_e}{\delta_n} = 1.9 \ln \frac{\Lambda\sqrt{n}}{T^2}, \tag{3}$$

where n is the particle density. As an example consider $T = 10^6$, $n = 10^{16}\ \mathrm{cm}^{-3}$ and $\ln\Lambda \sim 10$ then $\delta_e/\delta_n \sim 0.002$. Thus for a discharge of dimensions say, $a = 1$ cm and $L = 100$ cm the assumptions (1), (5) and (10) are compatible. Of course, in the vicinity of the electrodes these assumptions cannot be expected to hold, but then it is extremely unlikely that any of the assumptions (1) to (11) will be valid in these regions.

These assumptions differ from those of KAUFMANN and FURTH [3] by the inclusion of radiation losses and the effect of a longitudinal magnetic field. In a later section the effect of including terms in the conduction equations relating to thermoelectric effects is investigated.

While the assumptions are very restrictive and probably mean that the results cannot be applied directly to any known experiment, it is hoped that the results may be of value in estimating the importance of axial conduction effects in a pinched discharge.

It is assumed that the electrodes are maintained at $T_0 = 0°$ K, but it can be shown that electrode temperatures up to 1300° K do not appreciably affect the results obtained. Solution of the conduction equation enables an analysis of the stabilised linear discharge to be made in terms of seven parameters, namely, the electron density in the unpinched discharge, n_0, the radius of the discharge tube, R, the length of the tube, $2L$, the initial axial magnetic field, B_0, two skin depth parameters λ, μ, and the potential fall, $2\varphi_0$, between the electrodes.

In order to determine the time required for the establishment of a thermal steady state in a conduction-cooled linear discharge, a rough numerical solution of the non-linear time dependent conduction equation was obtained.* A pulse of 10^6 ° K was put on at the centre of the tube and maintained, whilst the electrodes were kept at 1300° K. For a tube of length 300 cm containing a fully ionised gas of 10^{14} particles per unit volume the time required to achieve the *thermal* steady state is of the order one microsecond, that is some 4 or 5 times greater than the relaxation time to be expected from evaluation of

$$\frac{L}{\sqrt{2\,kT_{\max}/m_e}}\,.$$

To establish pressure equilibrium in deuterium gas requires a time of about one order of magnitude greater than that for the thermal steady state to be set up. Thus it appears that for a slow linear pinch, when loss processes other than bremsstrahlung radiation and axial conduction are insignificant, the steady-state problem is of genuine interest.

In Section 2 the equations appropriate to the steady state are written down and the conduction equation solved. The results obtained are discussed in Section 3. The significance of thermoelectric effects is considered in Section 4. Finally, in Section 5 there is a general discussion of the results including a consideration of the regime in which they may be applicable.

Electromagnetic units are used throughout except for the electronic charge e which is expressed in electrostatic units, and except where otherwise indicated.

2. The steady-state problem

The formulae taken for electrical resistivity η, thermal conductivity K, and the amount of bremsstrahlung radiation ε, emitted per unit time per unit volume of plasma, are respectively, see SPITZER [7],

$$\left.\begin{aligned} \eta &= \eta_0\, T^{-\frac{3}{2}} \\ K &= K_0\, T^{\frac{5}{2}} \\ \varepsilon &= \varepsilon_0\, n^2\, T^{\frac{1}{2}} \end{aligned}\right\} \tag{4}$$

with

$$\eta_0 = 6.53 \times 10^{12} \ln\Lambda \ \mathrm{emu}\ (°\,\mathrm{K})^{\frac{3}{2}}$$

$$\varepsilon_0 = 1.42 \times 10^{-27}\ \mathrm{erg\ cm^3\ sec^{-1}}\ (°\,\mathrm{K})^{-\frac{1}{2}}.$$

The value for K_0 is given in Section 3. Since $\ln\Lambda$ varies only slowly with the particle density (measured along the axis of the tube) n, and the absolute temperature T, this dependence is neglected.

In the presence of a magnetic field both the thermal and electrical conductivities perpendicular to the field are reduced. It is possible therefore that the assumption of uniform temperature across the discharge might need refinement in order to deal with strong fields. However, for convenience we assume the uniform model to be appropriate both in the presence and absence of a magnetic field.

The axial electric field E_z is related to the electrostatic potential φ by the familiar form,

$$E_z = -\frac{\mathrm{d}\varphi}{\mathrm{d}x}, \tag{5}$$

x being the distance along the discharge measured from the anode. Other equations connecting the various axial electrical quantities are

$$E_z = j_z\, \eta \tag{6}$$

and

$$\frac{\mathrm{d}j_z}{\mathrm{d}x} = 0\,. \tag{7}$$

Since the axial magnetic field B_z is assumed completely trapped, it can be determined from

$$B_z = B_0\,(\pi R^2/A)\,, \tag{8}$$

where A is the cross-sectional area of the pinch.

* This numerical solution was obtained by M. R. WETHERFIELD.

The pinch relation of Bennett [6] modified for the inclusion of an internal axial magnetic field is

$$I_z^2 = \lambda^2 A^2 j_z^2 = 4\, N k T + \frac{B_0^2 A}{4\pi}\left(\frac{\pi R^2}{A}\right)2\,, \tag{9}$$

where N is the number of electrons (or ions) per unit length of discharge.

The discontinuity in the B_z field at the surface of the pinch gives rise to an azimuthal current of density j_θ flowing in a thin skin of width δ cm, such that

$$j_\theta = \frac{B_z}{4\pi\delta}\,. \tag{10}$$

If a is the radius of the pinch and noting that for $\delta \ll a$

$$\mu = \frac{2\pi a \delta}{A}\,, \tag{11}$$

then the heat conduction equation to be solved is

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{K\,\mathrm{d}T}{\mathrm{d}x}\right) = \varepsilon_0\, n^2\, T^{\frac{1}{2}} - \lambda E_z j_z - \mu j_\theta^2 \eta. \tag{12}$$

Using the boundary conditions $\varphi = 0$, $T = T_{\mathrm{m}}$ when $\mathrm{d}T/\mathrm{d}x = 0$, Eq. (12) may be integrated to give

$$\varphi^2 = \frac{\eta_0 K_0}{\lambda \beta(j_z)}\, T_{\mathrm{m}}^2 \left\{1 - \left(\frac{T}{T_{\mathrm{m}}}\right)^2\right\}, \tag{13}$$

where

$$\beta(j_z) = 1 - \frac{\lambda I_z^2}{I_0^2}\left(1 - \frac{\pi B_0^2 R^4}{4\lambda^2 A^3 j_z^2}\right)^2 + \frac{\mu}{\lambda}\cdot\frac{\pi B_0^2 R^4}{4\mu^2 A^3 j_z^2}\,, \tag{14}$$

and I_0, a critical current analogous to that arising in the work of Pease [8], is given by

$$I_0 = \left(\frac{16\,\eta_0 K_0^2}{\varepsilon_0}\right)^{\frac{1}{2}} = 1.44 \times 10^6 \text{ amp.} \tag{15}$$

In a practical case T_0, the temperature of an electrode, is of order 1000° K while Eq. (13) would not be expected to hold for $T_{\mathrm{m}} < 10^5$ ° K, as the level of ionisation would be inadequate. Thus when the electrodes are maintained at $\pm\varphi_0$ a term of order $(T_0/T_{\mathrm{m}})^2$ can be neglected. It can be shown that allowing $T_0 \to 0$ does not lead to any mathematical difficulty on account of the singularity in n.

Using the boundary conditions $\mathrm{d}T/\mathrm{d}x = 0$ at $T = T_{\mathrm{m}}$, $T = 0$ at $x = 0$, Eq. (12) can be made to give

$$C_2 \frac{x}{L} = \int_0^{\sin^{-1} T/T_{\mathrm{m}}} \sin^{\frac{5}{2}}\theta\, \mathrm{d}\theta\,, \tag{16}$$

where

$$C_2 = \eta_0\, T_{\mathrm{m}}^{-\frac{3}{2}}\, j_z \frac{L}{\varphi_0}\,, \tag{17}$$

thus exhibiting T/T_{m} as a universal function of x/L, that is

$$\frac{T}{T_{\mathrm{m}}} = F\left(\frac{x}{L}\right). \tag{18}$$

$F(x/L)$ has been plotted by Kaufman and Furth [3].

Since $T = 0$ when $x = 2L$ it is simple to show that

$$C_2 = \frac{(\pi)^{\frac{1}{2}}}{2}\,\frac{\Gamma(7/4)}{\Gamma(9/4)} = 0.719 = \frac{\eta_0\, T_{\mathrm{m}}^{-\frac{3}{2}}\, j_z}{\varphi_0/L}\,. \tag{19}$$

Equation (19) shows the current density in terms of the average potential gradient and maximum temperature. C_2 appears as a correction factor to the current density to be expected from the supposition of a uniform temperature throughout the tube.

Since the pressure throughout the length of the discharge is assumed to be constant,

$$N T = N_{\mathrm{m}}\, T_{\mathrm{m}}\,. \tag{20}$$

N_{m}, the number of particles per unit length at the centre of the tube, can be expressed in terms of the initial particle density, since assuming complete sweep-up of gas, the average number of particles per cm. length of discharge before constriction must be equal to that during constriction.

Using Eqs. (16), (18), (20),

$$N_{\mathrm{m}} = \frac{\pi R^2 n_0}{C_1}\,, \tag{21}$$

where

$$C_1 = \int_0^1 \frac{\mathrm{d}y}{F(y)} = \frac{\Gamma(5/4)\,\Gamma(9/4)}{\{\Gamma(7/4)\}^2} \sim 1.22\,. \tag{22}$$

C_1 is the factor by which the line density of the particles is reduced from its original value at the point of maximum temperature.

Using Eqs. (19) and (21), with the pinch relation, Eq. (9), for $T = T_{\mathrm{m}}$, leads to the cubic equation

$$Z^3 - \left(\frac{B_{\mathrm{c}}}{B_0}\right)^{\frac{4}{3}} Z - 1 = 0\,, \tag{23}$$

where

$$Z = A\, j_z^{\frac{2}{3}} \left(\frac{4\lambda^2}{\pi R^4 B_0^2}\right)^{\frac{1}{3}}, \tag{24}$$

and the critical field B_{c}, is given by

$$B_{\mathrm{c}} = \left(\frac{4\pi n_0 k}{C_1}\right)^{\frac{3}{4}} \cdot \left(\frac{4 L \eta_0}{\pi C_2 \varphi_0 \lambda R}\right)^{\frac{1}{2}}. \tag{25}$$

As an example for $\pi R^2 n_0 = 10^{16}$ particles/cm, $\varphi_0 = 4$ kV, $L = 225$ cm, $R = 20$ cm, $\lambda = 0.3$ then $B_{\mathrm{c}} \sim 4.4$ gauss, so that in most cases of interest it is likely that $B_0 \gg B_{\mathrm{c}}$.

After some elementary algebra one can derive the following result: —

$$\frac{c k T_{\mathrm{m}}}{e \lambda^{\frac{1}{2}} \varphi_0} = \frac{-s_1 + (s_1^2 + 4 s_2)^{\frac{1}{2}}}{2} = G(s_1)\,, \tag{26}$$

where

$$s_1 = \frac{4\pi R^2 n_0 c k^2 \varphi_0 \lambda^{\frac{2}{3}}}{C_1 I_0^2 e \eta_0 K_0}\left(1 - \frac{1}{Z^3}\right) \tag{27}$$

and

$$s_2 = \frac{C_2 k^2}{\eta_0 K_0 e^2}\left(1 + \frac{\lambda}{\mu}\cdot\frac{1}{Z^3}\right) = 0.618\left(1 + \frac{\lambda}{\mu}\cdot\frac{1}{Z^3}\right). \tag{28}$$

Further algebra leads to the following results for I_z^2 and A, respectively,

$$I_z^2 = 4\pi R^2 n_0 C_1^{-1} k T_{\mathrm{m}} Z^2 \left(\frac{B_0}{B_{\mathrm{c}}}\right)^{\frac{4}{3}} \tag{29}$$

$$A = \frac{Z R^2 C_1 B_0^{\frac{2}{3}} B_{\mathrm{c}}^{\frac{4}{3}}}{16 n_0 k T_{\mathrm{m}}} . \tag{30}$$

If Q_{c} is the heat dissipated through conduction via the electrodes and Q_{R} that lost by bremsstrahlung, then it can be shown that

$$\frac{Q_{\mathrm{R}}}{Q_{\mathrm{c}}} = \frac{I_z^2}{I_0^2 \left(\frac{B_0}{B_{\mathrm{c}}}\right)^{\frac{8}{3}} \left(\frac{Z^4}{\lambda} + \frac{Z}{\mu}\right) - I_z^2} . \tag{31}$$

This result indicates that for a sufficiently large axial magnetic field the radiation loss will be negligible compared with the thermal conduction loss except when unusually high currents flow. This effect is due to the "blowing out" of the discharge by the axial field trapped in the plasma. Numerical examples illustrating the above results are given in the following Section.

3. Examination of results

According to Spitzer [7] when no current flows the effective thermal conductivity is reduced due to thermoelectric effects. So far we have ignored these effects, but we shall see in Section 4, where a simplified model incorporating these effects is treated, that the reduced conductivity is appropriate in the present model. The appropriate value for K_0 is thus

$$K_0 = 1.84 \times 10^{-5} (\ln \Lambda)^{-1} \,\mathrm{erg\,sec^{-1}\,cm^{-1}} \,(^\circ\mathrm{K})^{-\frac{7}{2}} . \tag{32}$$

In all calculations a mean value for $\ln \Lambda$ equal to 15 is adopted.

We have seen in Section 2 that a typical value for B_{c} is of the order 4.4 gauss so that usually for a stabilised discharge $B_0 \gg B_{\mathrm{c}}$. In these circumstances the Eq. (23) for Z can be solved approximately to give

$$Z \sim 1 - \frac{1}{3}\left(\frac{B_{\mathrm{c}}}{B_0}\right)^{\frac{4}{3}} . \tag{33}$$

Eliminating Z between Eqs. (27) and (33) we obtain the following expression for s_1: —

$$s_1 \sim \left(\frac{B_{\mathrm{c}}}{B_0}\right)^{\frac{4}{3}} \frac{4\pi R^2 n_0 c k^2 \varphi_0 \lambda^{\frac{3}{2}}}{C_1 I_0^2 e \eta_0 K_0} . \tag{34}$$

This may be written

$$s_1 \sim 0.783 \,(\pi R^2 n_0)\, \lambda^{\frac{3}{2}} V \times 10^{-22} \left(\frac{B_{\mathrm{c}}}{B_0}\right)^{\frac{4}{3}} , \tag{35}$$

where V is the voltage across the tube.

The maximum temperature T_{m} is given by Eq. (26). This may be simplified if $s_1 \ll \sqrt{s_2}$, that is when the discharge is primarily cooled by conduction,

$$k T_{\mathrm{m}} = \left(\frac{e}{c}\right) \varphi_0 \lambda^{\frac{1}{2}} \left(\sqrt{s_2} - \frac{s_1}{2}\right) . \tag{36}$$

The first term of the bracket is that which would arise if thermal conduction were the only heat loss; the second is a correction due to radiation losses. In the examples which follow we take $\lambda \sim \mu$ and since $B_{\mathrm{c}} \ll B_0$, by Eq. (33), we may take $Z \sim 1$. As a particular example we may consider the following conditions: —

$$n_0 = 10^{13}\,\mathrm{cm^{-3}},\ \varphi_0 = 1\,\mathrm{kV},\ \lambda = 0.3,\ B_0 = 1000\,\mathrm{gauss} \qquad L = 180\,\mathrm{cm} \quad R = 45\,\mathrm{cm} . \tag{37}$$

The dimensions L and R, for the sake of definiteness. are those occurring in "Columbus T2" (see Baker *et al* [9]. By Eq. (25) this gives $B_{\mathrm{c}} \sim 6.3$ gauss; also Eq. (35) gives $s_1 \sim 1.9 \times 10^{-6}$. The maximum temperature T_{m} can be determined from Eq. (36) and is

$$T_{\mathrm{m}} \sim 7.1 \times 10^6 \,{}^\circ\mathrm{K} . \tag{38}$$

The current I_z is of the order 4.2×10^6 amp and A, the cross-sectional area of the pinch, is of the order 18 sq. cms.

Applying Eq. (31) to the above example we find that $Q_{\mathrm{R}}/Q_{\mathrm{c}} \sim 1.2 \times 10^{-6}$ indicating that radiation losses are negligible.

Consider, as second example, the following conditions: —

$$n_0 = 10^{14}\,\mathrm{cm^{-3}},\ \varphi_0 = 10\,\mathrm{kV},\ \lambda = 0.3,\ B_0 = 125\,\mathrm{gauss} \qquad L = 180\,\mathrm{cm}, \quad R = 45\,\mathrm{cm} . \tag{39}$$

Under these conditions $B_{\mathrm{c}} \sim 11.1$ gauss and $s_1 \sim .003$, thus

$$T_{\mathrm{m}} \sim 7.1 \times 10^7 \,{}^\circ\mathrm{K}, \quad I_z \sim 7.3 \times 10^6\,\mathrm{amp},$$

the last following from Eq. (29). The results arising for no axial field present are also obtainable from the previous expressions. The general expression for s_1 becomes

$$s_1 = \frac{4\pi R^2 n_0 c k^2 \varphi_0 \lambda^{\frac{3}{2}}}{C_1 I_0^2 e \eta_0 K_0} = 0.783\,(\pi R^2 n_0)\,\lambda^{\frac{3}{2}} V \cdot 10^{-22} . \tag{40}$$

From Eq. (36) we see that the presence of the magnetic field diminishes the effect of radiation loss. ($s_1 \ll \sqrt{s_2}$). Considering the conditions quoted in the second example above but with no magnetic field, it is found that

$$T_{\mathrm{m}} \sim 4.7 \times 10^7 \,{}^\circ\mathrm{K} . \tag{41}$$

We see that the effect of the magnetic field is to produce a negligible rise in temperature.

Using the same conditions, but with no magnetic field applied, I_z is found to be of the order 1.2×10^6 amp. Thus we see that the presence of the magnetic field leads to an increase in I_z by a factor of order 6.

The second example considered was selected in order to demonstrate the conditions under which radiation losses begin to become significant and, in fact, the value $\pi R^2 n_0 \sim 10^{18}\,\mathrm{cm^{-1}}$ is somewhat high for a practical discharge. A value $10^{16}\,\mathrm{cm^{-1}}$ might be more realistic, but in these circumstances radiation

losses are insignificant and the term s_1 in Eq. (36) may be neglected.

Equation (26) may also be simplified if the discharge is primarily cooled by radiation, that is, if $s_1^2 \gg s_2$. In this case

$$G(s_1) \sim \frac{s_2}{s_1} - \frac{s_2^2}{s_1^3} \tag{42}$$

and

$$k\,T_m = (e/c)\,\varphi_0\,\lambda^{\frac{1}{2}}\left\{\frac{s_2}{s_1} - \frac{s_2^2}{s_1^3}\right\}. \tag{43}$$

The current I_z is given, using Eq. (29), by the expression

$$I_z^2 = \frac{I_0^2}{\lambda} Z^4 \left(\frac{B_0}{B_c}\right)^{\frac{8}{3}} \left(1 - \frac{s_2}{s_1^2}\right). \tag{44}$$

As before, when $B_0 \gg B_c$, then $Z \sim 1$ and when $B_c \gg B_0$, then $Z^4(B_0/B_c)^{\frac{8}{3}} \sim 1$.

4. Thermoelectric effects

So far the existence of thermoelectric effects has been ignored in consideration of the discharge. These effects produce a heat flow in the opposite direction to an applied electric field, and an additional current in the direction of a temperature gradient, the equations controlling the heat flow and current being given by SPITZER [7] as

$$j = \frac{1}{\eta} E + \alpha \frac{\mathrm{d}T}{\mathrm{d}x} \tag{45}$$

$$Q = -\beta E - K' \frac{\mathrm{d}T}{\mathrm{d}x}. \tag{46}$$

In these equations, it is assumed that the heat flow Q, current density j, electric field E, and temperature gradient are all in the axial direction. α and β are given by SPITZER and HARM [10]. In the skin of the discharge Eqs. (45) and (46) hold without modification. In the main body of the discharge as before, we assume that no current flows due to inductive effects. Ignoring dependence on T through $\ln \Lambda$, α and β have the temperature dependence

$$\left.\begin{aligned} \alpha &= \alpha_0 T^{\frac{3}{2}} \\ \beta &= \beta_0 T^{\frac{5}{2}} \end{aligned}\right\}. \tag{47}$$

$$K' = K_0' T^{\frac{5}{2}}. \tag{48}$$

K' occurring in Eq. (46) is, of course, the thermal conductivity unmodified for thermoelectric effects.

Making use of Eqs. (45) to (48), we have in the skin

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = -\frac{\mathrm{d}}{\mathrm{d}x}\left\{K_0\, T^{\frac{5}{2}} \frac{\mathrm{d}T}{\mathrm{d}x}\right\} - \frac{\mathrm{d}}{\mathrm{d}x}(\beta_0\, \eta_0\, j_z\, T), \tag{49}$$

where

$$K_0 = K_0'\left(1 - \frac{\alpha_0 \beta_0 \eta_0}{K_0'}\right). \tag{50}$$

Whilst for the main body, we have

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = -\frac{\mathrm{d}}{\mathrm{d}x}\left(K_0\, T^{\frac{5}{2}} \frac{\mathrm{d}T}{\mathrm{d}x}\right). \tag{51}$$

To proceed further we interpret Q as an average heat flux by adding Eqs. (49) and (51) with weights λ and $1-\lambda$ respectively. We equate $\mathrm{d}Q/\mathrm{d}x$ to the joule heating in the plasma, for simplicity neglecting any losses due to bremsstrahlung radiation.

We thus obtain

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(K_0\, T^{\frac{5}{2}} \frac{\mathrm{d}T}{\mathrm{d}x}\right) + \lambda \beta_0\, \eta_0\, j_z \frac{\mathrm{d}T}{\mathrm{d}x} + \lambda E_z j_z + \mu j_0^2 \eta = 0. \tag{52}$$

In obtaining Eq. (52) we have also used the continuity equation $\mathrm{d}j_z/\mathrm{d}x = 0$. Substituting for E using Eq. (45), Eq. (52) reduces to

$$T^{\frac{3}{2}} \frac{\mathrm{d}}{\mathrm{d}x}\left(T^{\frac{5}{2}} \frac{\mathrm{d}T}{\mathrm{d}x}\right) + a\, T^{\frac{3}{2}} \frac{\mathrm{d}T}{\mathrm{d}x} + b = 0. \tag{53}$$

with

$$\left.\begin{aligned} a &= \lambda\, \eta_0\, j_z (\beta_0 - \alpha_0)/K_0 \\ b &= \frac{\lambda}{K_0} \eta_0\, j_z^2 \left\{1 + \frac{\lambda}{\mu} \frac{1}{Z^3}\right\} \end{aligned}\right\}. \tag{54}$$

The substitution in Eq. (53)

$$p = T^{\frac{3}{2}} \frac{\mathrm{d}T}{\mathrm{d}x} \tag{55}$$

leads to

$$p \frac{\mathrm{d}}{\mathrm{d}T}(p\,T) + a\,p + b = 0, \tag{56}$$

which can be integrated in the form

$$\ln T + \frac{1}{2}\ln(p^2 + a\,p + b) - \frac{a}{2}\int^{p} \frac{dp'}{p'^2 + a p' + b} = D \tag{57}$$

where D is a constant.

It can be proved using the results of SPITZER and HARM [10] that $4b - a^2 \geq 0$ for all values of λ between 0 and 1.*

Hence

$$q = +(4\,b - a^2)^{\frac{1}{2}} \tag{58}$$

is real; and Eq. (57) may be integrated to give

$$\frac{1}{2}\ln(p^2 + a\,p + b) - (a/q)\tan^{-1}\frac{2\,p + a}{q} + \ln T = D. \tag{59}$$

The constant D may be fixed from the boundary conditions $T = T_m$ when $p = T^{\frac{3}{2}} \mathrm{d}T/\mathrm{d}x = 0$, giving

$$D = \ln T_m - (a/q)\tan^{-1}(a/q) + \frac{1}{2}\ln b. \tag{60}$$

* Comparison of the present work with similar but independent work of M. G. HAINES [11] has revealed that use of the transport coefficients derived by W. MARSHALL [12] leads to $4b - a^2 < 0$, and this implies an inward heat flux at the cathode. Thus there appears to be a discrepancy between the Marshall and the Spitzer and Harm transport coefficients. As yet, the source of this discrepancy is unexplained.

T_m can be determined by considering the behaviour of Eq. (59) in the vicinity of the electrodes where $T \to 0$, obtaining the total heat flux outwards from the tube and equating this to the electrical power dissipated. Since at the electrodes, the heat flow $Q = -K_0 T^{\frac{5}{2}} \mathrm{d}T/\mathrm{d}x$ remains finite, $p = T^{\frac{3}{2}} \mathrm{d}T/\mathrm{d}x$ must tend to $+\infty$ at one electrode and $-\infty$ at the other. At the first electrode we have

$$\ln pT = D + \frac{a}{q} \cdot \frac{\pi}{2}, \qquad (61)$$

and at the second electrode

$$\ln(-pT) = D - \frac{a}{q} \cdot \frac{\pi}{2}. \qquad (62)$$

The total heat dissipated through conduction is thus

$$2 K_0 e^D \cosh\left(\frac{a}{q} \cdot \frac{\pi}{2}\right) = 2 \lambda \varphi_0 j_z \left(1 + \frac{\lambda}{\mu} \cdot \frac{1}{Z^3}\right), \qquad (63)$$

which simplifies to

$$T_m = \frac{e^{r \tan^{-1} r}}{\cosh r \pi/2} \cdot \sqrt{s_2} \, \frac{e}{c} \, \frac{\varphi_0 \lambda^{\frac{1}{2}}}{k}, \qquad (64)$$

where $r = a/q$.

Thus the results agree with those for the situation in which thermoelectric effects are neglected with the exception of the factor $[\exp(r \tan^{-1} r)]/\cosh(\pi r/2)$ which tends to $2/e$ for $r \to \infty$ and to 1 as $r \to 0$. Substitution of a value 0.3 for λ, and values of α_0, β_0 from Spitzer and Harm [10] leads to a value of $r \sim 0.41$ when $[\exp(r \tan^{-1} r)]/\cosh(\pi r/2)$ becomes 0.97 which is negligibly different from its limiting value 1.0.

5. Discussion

Before making any statement of the general conclusions reached it is important to consider some of the underlying restrictions.

First, the model only applies fully when equilibrium is attained between ions and electrons. Before this state is reached, however, the model will still apply approximately so long as we consider the temperatures involved as electron temperatures.

Second, the only loss mechanisms we have considered are bremsstrahlung radiation and axial conduction. Many other processes are in fact possible and may be dominant under suitable conditions, see, e.g. Tuck [13].

Third, it must be remembered that all previous deductions have been based upon macroscopic equations which are dependent for their validity upon the condition

$$\left| \frac{l}{X} \frac{\mathrm{d}X}{\mathrm{d}x} \right| \ll 1, \qquad (65)$$

where X is any macroscopic variable and l the mean free path. For the macroscopic variable $X = T$ it is a simple matter to show that the above condition is equivalent to the fact that the rate of transport of heat energy must be very much less than the velocity of sound in the medium.

Also this condition, Eq. (65), is a necessary condition for an approximately Maxwellian distribution of velocities to be maintained locally; it should be remembered however that the well-known "Langmuir Paradox" deals precisely with the situation that a Maxwellian distribution of electron velocities is found experimentally to exist in situations where Eq. (65) fails to hold, see Gabor, Ash and Dracott [14]. Thus the condition may be too stringent.

In general, the condition of Eq. (65) cannot be satisfied throughout the entire length of the tube. Thus for temperatures sufficiently small, i.e. $T < T'$, the above condition will be violated. Our previous results have been derived assuming $T_m \gg T'' \sim 10^5$ °K, where T'' is that temperature at which a high degree of ionisation is possible.

If $T' < T''$ the violation of Eq. (65) will only occur in regions near the electrodes where ionisation is incomplete and the uncertainty introduced is no greater than through the neglect of sheath effects. Even if this condition is not satisfied, providing that $T_m \gg T'$, then Eq. (65) will be satisfied over the major part of the length of the tube and again it might be anticipated that the formulae will hold unless conditions are dominated by the neglected effects in the neighbourhood of the electrodes.

As regards the importance of thermoelectric phenomena, when the discharge is principally conduction cooled (as is likely in stable linear pinched discharges of current interest) the chief thermoelectric effect is to replace the unmodified thermal conductivity K' by the modified K_0 and to slightly reduce the temperature reached for a given voltage across the tube. In addition the original symmetrical distribution of temperature and density in the tube is replaced by an asymmetrical one. The precise form of the distribution and the modification to the formulae for current and discharge cross-section would require an integration of Eq. (59) but it seems unlikely that previous results will be changed drastically.

Within these limitations, we may draw certain conclusions. First, in nearly all circumstances the main loss from a linear pinched discharge is likely to be from thermal conduction. In order that this should be so the following condition must be satisfied:

$$\frac{1}{Z^4} \left(\frac{B_c}{B_0}\right)^{\frac{8}{3}} \left(\frac{4 \pi R^2 n_0 c k^2 \varphi_0 \lambda^{\frac{3}{2}}}{c_1 I_0^2 e \eta_0 K_0}\right)^2 \ll s_2 \sim 0.62. \qquad (66)$$

In these circumstances the highest temperature that can be reached, taking the thermoelectric effect to be negligible, is

$$T_m = 4.6 \, V \sqrt{\lambda (1 + \lambda/\mu)} \, 10^3 \,{}^\circ\mathrm{K}, \qquad (67)$$

V being the voltage across the tube.

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LONGITUDINAL OSCILLATIONS IN A NEUTRALIZED ELECTRON BEAM WITH A BOUNDARY

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Longitudinal oscillation of a neutralized electron beam with a boundary in a strong axial magnetic field is discussed. The beam is assumed to be cylindrical and to be confined in a conducting cylinder, or to be planar and to be held between two conducting plates. A sufficient and, in some cases, approximately necessary condition for stability is obtained. To be stable, the beam current should be below a certain value which depends on the electron velocity, the ratio of electron to ion mass and the geometrical dimensions of the beam and the conductor. The validity of the approximation is also studied.

1. Introduction

In this paper we shall consider longitudinal oscillations in cylindrical and planar electron streams neutralized by positive ions in the absence of collisions. Many authors have discussed similar problems for the case of infinite streams, taking into account the effect of thermal motions in the Boltzmann equations [1, 2, 3]. Their results show that, unless the energy of the thermal motions is comparable to that of the ordered motions, the system is unstable with respect to oscillation. This self-excited oscillation may prevent production of electron beams of high intensity although the oscillation may also be advantageous from the thermonuclear point of view in that oscillations of ions are stimulated and their random energy raised.

The case of bounded streams may be of particular interest as the situation is really true in plasma betatrons [4], high intensity accelerators of neutralized electron beams, in the some thermonuclear machines.

For tractability we shall hereafter neglect the random motions of electrons and ions and shall assume that the beam holds a strong axial magnetic field within it and that the magnetic field produced by the beam itself is negligible. In what follows, a stability criterion will be derived and discussed. The criterion states that the longitudinal oscillation can not be excited spontaneously if the beam current is below some critical value which depends on experimental conditions. The validity of the above assumptions will also be discussed.

2. Dispersion relations for the cylindrical case

We consider a uniform cylindrical beam of neutralized electrons. The beam has a radius d, is streaming along a strong axial magnetic field and is enclosed in a conducting cylinder of radius D. The intervening space ($d \leq r \leq D$) is assumed to be vacuum. Using cylindrical coordinates coaxial with the cylindrical beam, we can find a stationary state of the neutralized beam. This state is characterized by the velocity $\mathbf{v}_{e0}$ (r, θ, z components $=0$, $v_{e\theta 0}$, v_0), the velocity of ions $\mathbf{v}_{i0}$ $(0, v_{i\theta 0}, 0)$, the partial pressure and the density of electrons p_{e0} and n_0, the partial pressure and the density of ions p_{i0} and n_0, the electric field $\mathbf{E}_0$ $(E_{r0}, 0, 0)$ and the magnetic field $\mathbf{B}_0$ $(0, B_{\theta 0}, 0)$.

In order to investigate the behaviour of the beam for small deviations from the stationary state we expand these quantities to the first order. The first order approximation for the electron velocity, the ion velocity, the density of electrons, the density of ions, the electric field and the magnetic field are to be denoted by

$$\mathbf{v}_e(v_{er}, v_{e\theta}, v_{ez}), \mathbf{v}_i(v_{ir}, v_{i\theta}, v_{iz}), n_e, n_i, \mathbf{E}(E_r, E_\theta, E_z), \mathbf{B}(B_r, B_\theta, B_z)$$

respectively. Some of the basic equations are closed in themselves, which are given (in MKS units) by

$$m_e \gamma^3 \left(\frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\right) v_{ez} = -e E_z \tag{1}$$

$$m_i \frac{\partial v_{iz}}{\partial t} = e E_z \tag{2}$$

$$\frac{\partial n_e}{\partial t} + v_0 \frac{\partial n_e}{\partial z} + n_0 \frac{\partial v_{ez}}{\partial z} = 0 \tag{3}$$

$$\frac{\partial n_i}{\partial t} + n_0 \frac{\partial v_{iz}}{\partial z} = 0 \tag{4}$$

$$\frac{\partial^2 E_z}{\partial z \partial r} - \frac{\partial^2 E_r}{\partial z^2} + \frac{1}{c^2} \frac{\partial^2 E_r}{\partial t^2} = 0 \tag{5}$$

$$\frac{\partial^2 E_r}{\partial z \partial r} + \frac{1}{r} \frac{\partial E_r}{\partial z} - \frac{\partial^2 E_r}{\partial r^2} - \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} = -\mu \frac{\partial i_z}{\partial t} \tag{6}$$

$$-e n_0 v_{ez} - e n_e v_0 + e n_0 v_{iz} = i_z \tag{7}$$

where we have assumed that the deviations are independent of θ. As for the case when they vary as $e^{im\theta}$ some discussions are given in Section 4. Here i_z is the z-component of the current of the first order and γ means, as usual, $(1-v_0^2/c^2)^{-\frac{1}{2}}$. μ and ε represent permeability and dielectric constant of vacuum.

In the right hand sides of Eqs. (1) and (2) the terms $-ev_{er} B_{\theta 0} + ev_{e\theta 0} B_r$ and $+ev_{ir} B_{\theta 0}$, respec-

tively, have been omitted in assuming that the magnetic self-field of the beam is negligible. This means that the pressure $p_{e0}+p_{i0}$ is also negligibly small. Consistently, we shall assume that the energy of the random motions of electrons and ions is much smaller than the energy of the unidirectional motion of electrons. Also in the left hand sides of Eqs. (3) and (4) we have dropped the terms $n_0\,\partial v_{er}/\partial r$ and $n_0\,\partial v_{ir}/\partial r$, respectively, keeping in mind that the axial magnetic field constrains the ions and the electrons to move along the lines of force.

Now we study a normal mode that varies with t and z as exp $(i\omega t - ikz)$. Substituting this into Eqs. (1) to (7) and rearranging, we obtain an equation for the amplitude of the axial component of the electric field,

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}E}{\mathrm{d}r}+k'^2 s^2 E_z=0\,, \tag{8}$$

where

$$s^2=\frac{\omega_{pe}{}^2\gamma^{-3}}{(\omega-kv_0)^2}+\frac{\omega_{pi}{}^2}{\omega^2}-1\,, \tag{9}$$

$$k'^2=k^2-(\omega/c)^2\,, \tag{10}$$

$$\omega_{pi}{}^2=\frac{n_0 e^2}{\varepsilon m_i}\,,\quad \omega_{pe}{}^2=\frac{n_0 e^2}{\varepsilon m_e}\,. \tag{11}$$

In vacuum ($d\le r\le D$) it follows

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}r\frac{\mathrm{d}E_z}{\mathrm{d}r}-k'^2 E_z=0\,. \tag{12}$$

The solution of Eq. (8) is $J_0(k'sr)$ and the solutions of Eq. (12) are linear combinations of $I_0(k'r)$ and $K_0(k'r)$. Furthermore, boundary conditions are

$$E_z(D)=0\,, \tag{13}$$

$$E_z(d+0)-E_z(d-0)=0\,, \tag{14}$$

$$\frac{\mathrm{d}E_z(d+0)}{\mathrm{d}r}-\frac{\mathrm{d}E_z(d-0)}{\mathrm{d}r}=0\,. \tag{15}$$

The first and the second conditions imply that parallel components of the electric field are continuous at the boundary; these can be generally derived from one of the Maxwell equations. The third condition is to be deduced from the consideration that in case of pure longitudinal oscillations there appears no surface current at the boundary ($r=d$) of the beam.

Finally, substituting the solutions of Eqs. (8) and (12) into Eqs. (13), (14) and (15), we obtain a dispersion relation for the normal mode. Introducing new parameters:

$$x=k'd\,,\ X=k'D\,,\ S=s\,x\,, \tag{16}$$

the dispersion relation becomes:

$$S\frac{J_1(S)}{J_0(S)}=x\frac{K_0(X)\,I_1(x)+I_0(X)\,K_1(x)}{I_0(X)\,K_0(x)-K_0(X)\,I_0(x)}\equiv\Psi\left(\frac{D}{d},kD\right). \tag{17}$$

Roots of Eq. (17) are many-valued and we denote them as $S(X/x,\ X;j)$, where the magnitude of the roots increases as $j=1,2,\ldots$ and so on, corresponding to the number of loops of E_z in the radial direction.

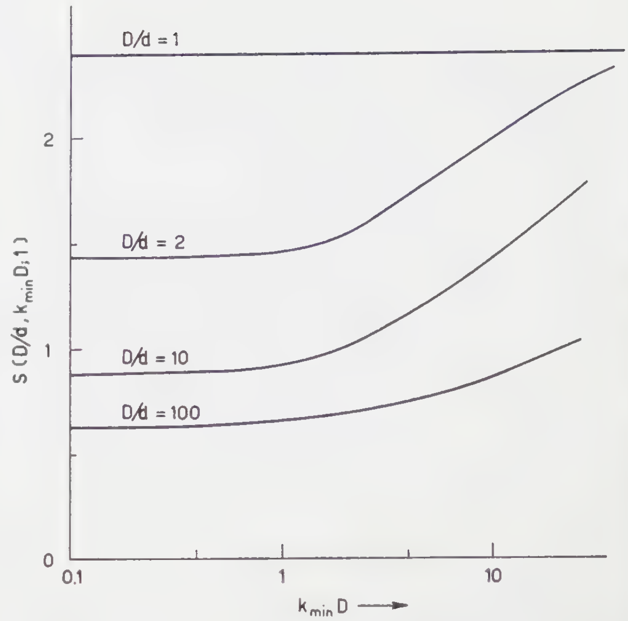

Fig. 1 $S(D/d,\ k_{\min}D;\ 1)$ versus $k_{\min}D$ for the cylindrical case. This characteristic parameter should be used to obtain stability conditions in Eq. (21).

$S(D/d,\ kD;1)$ with $k=k_{\min}$ (see discussion of Eqs. (20) and (21)) is shown in Fig. 1 for the cylindrical case and in Fig. 5 for the planar case (which we shall treat in Section 3). The curves are shown as a function of kD, regarding D/d as a parameter. The values chosen for D/d are 1, 2, 10 and 100. kD can range from 0.1 to over 20. In the cylindrical case $S(D/d,\ kD;1)$ is a slowly varying function of kD. It can be seen from Fig. 1 that the largest value of d(log S)/d(log kD) is about 0.2. The dependence becomes stronger in the case of the planar beam, see discussion in Section 3. Nevertheless the maximum value of d (log S)/d (log kD) for a wide range of parameters is 0.5 or less. Therefore it follows from Eq. (16) that, for a fixed value of D/d, the quantity s, which is $S(D/d,\ k'D;1)/kd$, is a monotonously decreasing function of kd because of the predominating dependence on the kd in the denominator.

The dispersion relation composed of Eqs. (9) and (17) can be solved by graphical manipulation as follows. We write down the dispersion relation in the form of

$$\frac{\omega_{pe}{}^2\gamma^{-3}}{(\omega-kv_0)^2}+\frac{\omega_{pi}{}^2}{\omega^2}=1+\frac{S^2(D/d,\ k'D;\ j)}{k'^2 d^2}\,. \tag{18}$$

$$(j=1,\ 2,\ldots)$$

We shall plot the both sides of the equation as functions of ω for fixed values of D/k, d and D. We have shown in Figs. 2 and 3 two typical figures; the former has four real roots (stable case) and the latter has two real roots and two (mutually conjugate) complex roots (unstable case). The critical case, when we have two single real roots and one double (real) root, defines a boundary that separates the stable region from the unstable region in parameter space. This

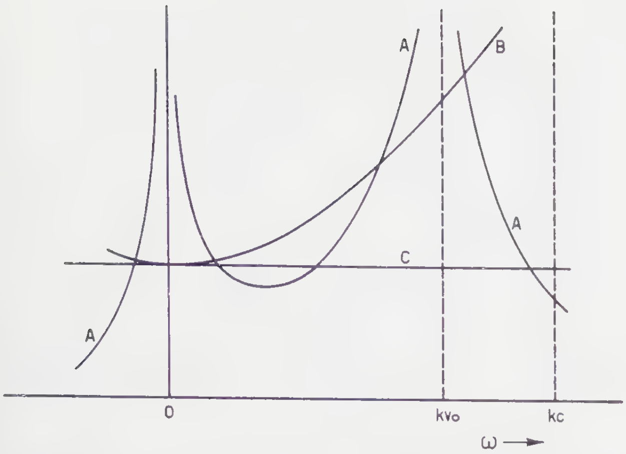

Fig. 2 Graphical method of solution is indicated schematically. The solutions are defined as the intersection points of the curves corresponding to both sides of the dispersion relation, Eq. (18). Curve A is a plot of the left hand side of Eq. (18), curve B the right hand side and curve C the right hand side with k' replaced by k [see Eq. (18a)]. In this figure there are four real roots (stable case).

can be obtained graphically without much trouble using Figs. 1 and 5 to obtain $S(D/d, k'D;1)$. Here we have taken $j=1$ because waves with higher values of j are more stable; this is so because in general $S(D/d, k'D; j>1) > S(D/d, k'D;1)$ holds.

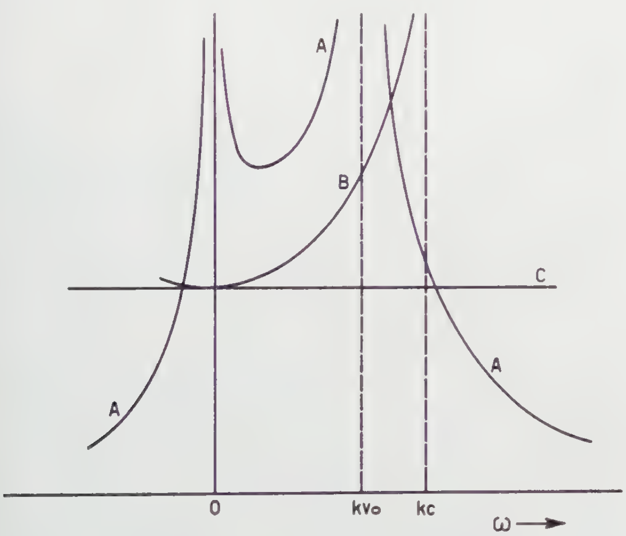

Fig. 3 Graphical method for an unstable case. The curves are labeled as described in the caption for Figure 2. There are two real roots as intersection points. In addition, there are two complex conjugate roots.

However, it would be quite convenient if we could have a simpler solution of the dispersion relation. Now we can derive a sufficient condition for the stability. As shown in Figs. 2 and 3, when k' in the right hand side of Eq. (18) is replaced by k, this side corresponds to a straight line parallel to the ω axis and is constantly below the original function $1+[S^2(D/d, k'D;1)/k'^2 d^2]$. Therefore if the equation

$$\frac{\omega_{\mathrm{pe}}^2\gamma^{-3}}{(\omega-kv_0)^2}+\frac{\omega_{\mathrm{pi}}^2}{\omega^2}=1+\frac{S^2(D/d,\ kD;\ 1)}{k^2 d^2} \tag{18a}$$

has four real roots, the original Eq. (18) also has four real roots. The sufficient condition can be derived if we note that the minimum of the left hand side as a function of ω is equal to

$$\left[\omega_{\mathrm{pe}}^{\frac{2}{3}}\gamma^{-1}+\omega_{\mathrm{pi}}^{\frac{2}{3}}\right]^3/k^2v_0^2$$

and

$$1+\frac{S^2(D/d,\ kD;\ 1)}{k^2d^2}\geq\frac{\left[\omega_{\mathrm{pe}}^{\frac{2}{3}}\gamma^{-1}+\omega_{\mathrm{pi}}^{\frac{2}{3}}\right]^3}{k^2v_0^2} \tag{19}$$

for all k.

In this connection, when $m_e\gamma^3 \ll m_i$,

$$\frac{\omega_c^2}{k_c^2c^2}=\frac{\beta^2\,\omega_{\mathrm{pi}}^{\frac{4}{3}}}{\left[\omega_{\mathrm{pe}}^{\frac{2}{3}}\gamma^{-1}+\omega_{\mathrm{pi}}^{\frac{2}{3}}\right]^2}$$

is a small quantity and the former stability condition can be regarded as an approximate necessary condition, where ω_c and k_c are the values associated with the equality sign in Eq. (19).

D and d are parameters that characterize an experimental device. So is $k_{\min}$ the minimum value of k, if we apply a periodic boundary condition in z. It turns out by numerical calculation that for fixed values of D and d, $\psi(D/d, kD)$ and consequently $S(D/d, kd;j)$ decrease monotonously as k decreases. Therefore the sufficient condition for the stability of the j-th mode becomes

$$1+\frac{S^2(D/d,\ k_{\min}D;\ j)}{k_{\min}^2d^2}\geq\frac{\left[\omega_{\mathrm{pe}}^{\frac{2}{3}}\gamma^{-1}+\omega_{\mathrm{pi}}^{\frac{2}{3}}\right]^2}{k^2v_0^2}. \tag{20}$$

Inequality (20) is satisfied when and only when inequality (20) for $j=1$ is satisfied. Introducing a dimensionless parameter, $\nu=(\pi d^2)(n_0 r_0)$ where r_0 is the classical radius of electron $e^2/4\pi\varepsilon m_e$, Eq. (17) for $j=1$ becomes

$$\nu\leq\frac{1}{4}[S^2(D/d,\ k_{\min}D;\ 1)+k_{\min}^2d^2]$$
$$\times\left[1-\frac{1}{\gamma^2}\right]\gamma^3\left[1+\left(\frac{m_e\gamma^3}{m_i}\right)^{\frac{1}{3}}\right]^{3}. \tag{21}$$

$S(D/d, k_{\min}D;1)$ as a function of $k_{\min}D$ for typical values of D/d is shown in Fig. 1. When $k_{\min}D$ approaches zero, $S(D/d, k_{\min}D;1)$ tends to a finite value $S(D/d, 0;1)$, as shown in Fig. 4. Furthermore $S(D/d, 0;1)$ has the following limiting properties,

$$S(D/d,\ 0;\ 1)\to 2.405[1-\log(D/d)],\ \{\log(D/d)\to 0\} \tag{22}$$

$$S(D/d,\ 0;\ 1)\to\left[\frac{2}{\log(D/d)}\right]^{\frac{1}{2}},\ \{\log(D/d)\to\infty\}. \tag{23}$$

Hence, the sufficient condition for stability for $k^2_{\min}D^2 \ll 1$ is

$$1.45[1-2\log(D/d)]\gamma^3\left(1-\frac{1}{\gamma^2}\right)\left[1+\left(\frac{m_e\gamma^3}{m_i}\right)^{\frac{1}{3}}\right]^{-3}\geq\nu \tag{24}$$

$$\log(D/d)\ll 1,$$

$$0.5\,[\log(D/d)]^{-1}\left(1-\frac{1}{\gamma^2}\right)\left[1+\left(\frac{m_e\gamma^3}{m_i}\right)^{\frac{1}{3}}\right]^{-3} \geq \nu \tag{25}$$

$$\log(D/d) \gg 1\,.$$

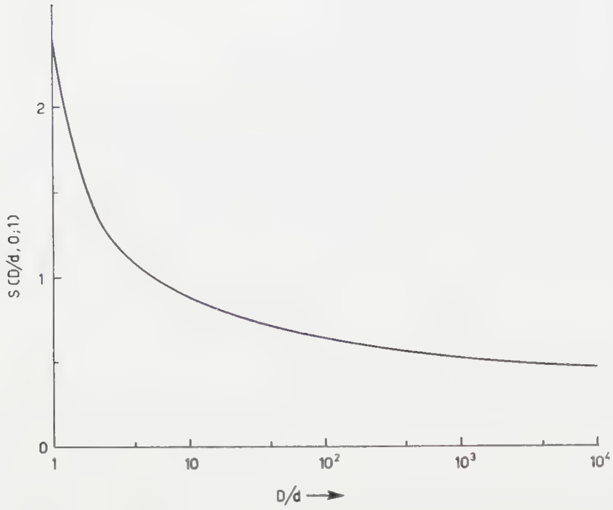

Fig. 4 $S(D/d,\ 0;1)$ versus D/d for the cylindrical case. This characteristic parameter should be used to obtain stability conditions in Eq. (21).

3. Dispersion relation for the planar case

For a planar neutralized electron beam of thickness $2d$ between two parallel conducting plates separated by $2D$, where $D>d$, a dispersion relation similar to that for the cylindrical case can be obtained:

$$S\tan S = x\cot(X-x)\,. \tag{26}$$

We can proceed as before and have only to present $S(D/d,\ k_{\min}D;1)$ and $S(D/d,\ 0;1)$ in Figs. 5 and 6

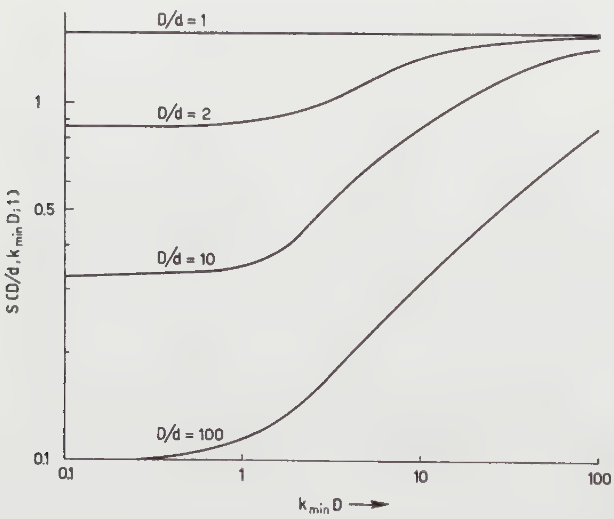

Fig. 5 $S(D/d,\ k_{\min}D;\ 1)$ versus $k_{\min}D$ for the planar case. This characteristic parameter should be used to obtain stability conditions in Eq. (21).

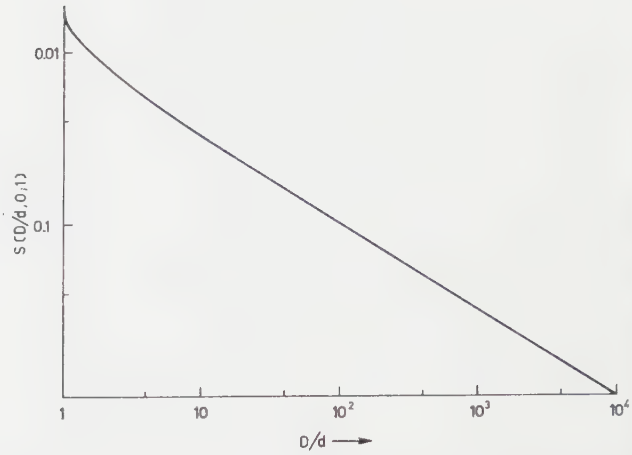

Fig. 6 $S(D/d,\ 0;\ 1)$ versus D/d for the planar case. This characteristic parameter should be used to obtain stability conditions in Eq. (21).

respectively. Eq. (18) holds good in this case and instead of Eqs. (19) and (20) we have, for $k_{\min}^2 D^2 \ll 1$:

$$0.62\{1-2[(D/d)-1]\}(1-\gamma^{-2})\gamma^3\left\{1+\left(\frac{m_e\gamma^3}{m_i}\right)^{\frac{1}{3}}\right\}^{-3} \geq \nu \tag{27}$$

$$(D/d)-1 \ll 1\,,$$

$$0.25\,(D/d)(1-\gamma^{-2})\gamma^3\left\{1+\left(\frac{m_e\gamma^3}{m_i}\right)^{\frac{1}{3}}\right\}^{-3} \geq \nu \tag{28}$$

$$(D/d) \gg 1\,.$$

4. Discussion

In Section 2 we assumed that the field $\mathbf{E}$ is uniform in the azimuthal direction. We can proceed as before when it varies as $\exp(im\theta)$. Also we may assume that the field changes in the direction parallel to the conductor plane and normal to the direction of the static magnetic field. Then similar dispersion relations can be obtained, but they are, as numerical calculations show, less restrictive conditions than Eq. (21).

The stability criterion Eq. (21), is not a very severe restriction on a device of usual scale when the beam is relativistic. As $\gamma^3[1+(m_e\gamma^3/m_i)^{1/3}]^{-3}$ is an increasing function of γ with a value between $[1+(m_e/m_i)^{1/3}]^{-3}$ and $(m_e/m_i)^{-1}$, an electron current of hundreds of thousands amperes can flow without causing the instability. When the beam is non-relativistic, the effect of the conducting wall can no longer stabilize the beam and ν decreases as $S^2\beta^2/4$.

In Section 2 we have neglected $-e(v_{er}B_{\theta 0}-v_{e\theta 0}B_r)$ in the right hand side of Eq. (1), $ev_{ir}B_{\theta 0}$ in the right hand side of Eq. (2), $n_0\,\partial v_{er}/\partial r$ in the left side of Eq. (3), and $n_0\,\partial v_{ir}/\partial r$ in the left side of Eq. (4). These simplifications, which are correct when the axial magnetic field is infinitely strong, have been convenient for us in extracting pure longitudinal oscillations and in studying the two-stream instability in a neutralized electron beam with a boundary. Without these assumptions the longitudinal oscilla-

tions would naturally couple with transverse oscillations and the resulting dispersion relation would be much more complicated. In this connection, the conclusion reached in Section 2 and Section 3 should be altered for frequencies in the vicinity of the frequencies of the ion and electron cyclotron waves.

It is difficult to express in a closed form how far the simplifications can be justified. Accordingly, we take two typical cases that can be treated analytically. The first case is when $k \to 0$ and the second is when $k = k_c$, where k_c is defined by Eq. (19). The former is significant for the reason that the two-stream amplification is more dangerous in the long-wave limit. Using new parameters,

$$p = \omega_{ce}/\omega_{pe} \tag{29}$$

$$q = (d/c)\,\omega_{pe} \tag{30}$$

$$\mu = m_e/m_i \tag{31}$$

$$b = B_{\theta 0}/B_{z0} \tag{32}$$

$$\omega_{ce} = (e/m_e)\,B_{z0} \tag{33}$$

and further assuming for simplicity

$$\frac{S^2}{k^2 d^2} \gg 1 \tag{34}$$

$$\frac{d^2 \omega^2_{pe}}{\gamma^3 S^2 v_0^2} = \frac{q^2}{\beta^2 \gamma^3 S^2} \ll 1 \tag{35}$$

$$\frac{d^2 \omega_{pi}^2}{S^2 v_0^2} = \frac{q^2 \mu}{\beta^2 S^2} \ll 1\,, \tag{36}$$

we obtain

$$\left|\frac{\partial v_{er}/\partial r}{\partial v_{ez}/\partial z}\right| \sim \left|\frac{1}{W}\left[\gamma\left(1 - \frac{\beta^2\gamma^4}{p^2\mu^2}\right) + \frac{\beta^3 \gamma^{\frac{7}{2}} q}{S}\left(1 + \frac{\gamma^2}{p^2\mu^2}\right)\right]\right| \ll 1 \tag{37}$$

$$\left|\frac{\partial v_{ir}/\partial r}{\partial v_{iz}/\partial z}\right| \sim \left|\frac{1}{W}\,\frac{\beta^2\gamma^2 S^2}{q^2\mu^2}\right| \ll 1 \tag{38}$$

$$\left|\frac{v_{e\theta 0} B_r}{E_z}\right| \sim \left|\frac{b\beta^3\gamma^2 p q\left(1 + \frac{\gamma^2}{p^2\mu^2} - \frac{\beta^2}{\gamma^4\mu^3 p^4}\right)}{WS\left(1 - \frac{\beta^2\gamma^2}{p^2\mu}\right)}\right| \ll 1 \tag{39}$$

$$\left|\frac{v_{er} B_{\theta 0}}{E_z}\right| \sim \left|\frac{b p}{\gamma^{3/2}}\,\frac{\partial v_{er}/\partial r}{\partial v_{ez}/\partial z}\right| \ll 1 \tag{40}$$

$$\left|\frac{v_{ir} B_{\theta 0}}{E_z}\right| \sim \left|\frac{b p q \mu}{\beta S}\,\frac{\partial v_{ir}/\partial r}{\partial v_{iz}/\partial z}\right| \ll 1 \tag{41}$$

$$W = p^2 - \frac{\beta^2\gamma^2}{\mu} - \frac{\beta^3\gamma^{\frac{7}{2}} q}{S p^2 \mu^2} + \frac{\beta^4\gamma^4 q^2}{S^2 p^2 \mu^2}\,. \tag{42}$$

These inequalities can be satisfied by an infinitely large p. It follows further that q, and therefore d, cannot be made arbitrarily small and that in this limit p^2 should be much larger than $\gamma^2/(q^2\mu^2)$.

In a contrasting case when $k^2 = k_c^2$, that is, when the system is near the critical thershold of stability, we obtain,

$$\left|\frac{\partial v_{er}/\partial r}{\partial v_{ez}/\partial z}\right| \sim \left|\frac{\beta^2\gamma^4 S^2}{p^2 q^2 V}\left\{1 - \frac{\beta^2\gamma}{p^2\mu^{5/3}}(1+\beta^2\gamma^2) - \mu^{\frac{1}{3}}\beta^4\gamma^3\right\}\right| \ll 1 \tag{43}$$

$$\left|\frac{\partial v_{ir}/\partial r}{\partial v_{iz}/\partial z}\right| \sim \left|\frac{\beta^2\gamma^2 S^2}{p^2 q^2 \mu^{4/3} V}\right| \ll 1 \tag{44}$$

$$\left|\frac{v_{e\theta 0} B_r}{E_z}\right| \sim \left|\frac{\beta^3\gamma^2 S}{p^3 q V}\left(1 - \beta^2\gamma^2 p^2 \mu^{\frac{1}{3}} - \frac{\beta^2\gamma^2}{\mu^{5/3}}\right)\right| \ll 1 \tag{45}$$

$$\left|\frac{v_{er} B_{\theta 0}}{E_z}\right| \sim \left|\frac{\gamma^3 b p q}{\beta S}\,\frac{\partial v_{er}/\partial r}{\partial z_{ez}/\partial z}\right| \ll 1 \tag{46}$$

$$V = 1 - \frac{\beta^4\gamma^3}{p^4\mu^{4/3}}(1 + \beta^2\gamma^2)\,, \tag{47}$$

where we have assumed $\gamma^{\frac{1}{3}}\mu \ll 1$, $p^2\mu^{\frac{4}{3}}\gamma \gg 1$ and $\beta\gamma^{\frac{3}{2}} S/q \ll 1$, in order to yield a brief result. These relations are again satisfied by an infinitely large p and do not hold when q is arbitrarily small.

Acknowledgement

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RADIATION FROM A MODULATED BEAM OF CHARGED PARTICLES PENETRATING A PLASMA IN A UNIFORM MAGNETIC FIELD

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The radiation from a density-modulated beam of ions, which penetrates a plasma perpendicular to a strong magnetic field $\mathbf{B}_0$, is investigated in two simplified situations: (a) the beam is an infinite plane parallel to $\mathbf{B}_0$, and (b) the beam is an infinite cylindrical surface parallel to $\mathbf{B}_0$, the radius of the cylinder being the gyroradius of the beam particles. This latter beam can be ideally constructed by injecting into a plasma a linear beam, modulated at a frequency which is an integral multiple of the gyrofrequency of the beam particles and incident in a direction which forms a very small angle with a plane perpendicular to $\mathbf{B}_0$.

In both situations some resonances of the Poynting vector are found. The resonance, which occurs when the modulation frequency is equal to the "ion-resonance" frequency, is specifically investigated, taking into account the finite electric conductivity of the plasma. It is shown that, under appropriate conditions, the beam-plasma interaction at this resonance becomes very strong.

1. Introduction

The radiation from an intensity-modulated beam of ions directed into a plasma in the presence of a strong magnetic field, constant in space and time, has recently been investigated by KIPPENHAHN and DE VRIES [1]. The investigation was made in connection with the problem of the injection and heating of a plasma. Kippenhahn and de Vries considered modulation frequencies sufficiently low, plane beams, and infinite electric conductivity of the plasma. They found that the energy loss from the beam is always negligible, except when the velocity of the beam particles is equal to the Alfvén velocity.

In the present paper we provide an extension which is useful in all ranges of frequencies and takes account of the curvature experienced by a two-dimensional beam in an external magnetic field. We show that the resonance of the kind found in [1] is practically destroyed by the beam curvature, but that other infinities occur in the Poynting vector at particular values of the modulation frequency. By taking into account the finite electric conductivity of the plasma, the peak value of the energy loss from the beam at the "ion-resonance" frequency is determined and briefly discussed. Such a peak value has, in many practical situations, a great order of magnitude and is a maximum when ω_p^2 is of the order of $\omega_i \omega_e$.

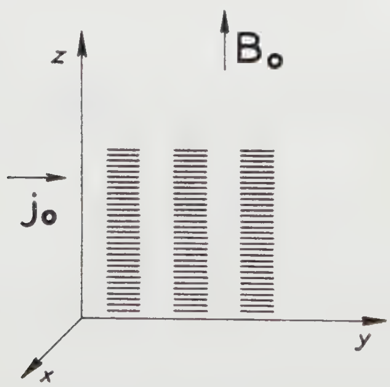

Fig. 1 The plane beam.

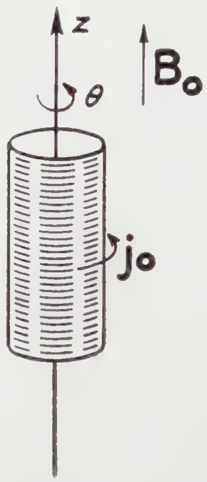

Fig. 2 The curved beam.

The beams here considered are:

i) an infinite plane parallel to the strong magnetic field $\mathbf{B}_0$, constant in space and time (Fig. 1).

ii) a cylindrical surface parallel to $\mathbf{B}_0$, whose radius is the gyroradius of the beam particles in $\mathbf{B}_0$ (Fig. 2).

We assume that all the ions in each beam travel perpendicularly to $\mathbf{B}_0$ with the same speed, that the velocity is not modified by the interaction with the plasma and that the plasma is infinitely extended.

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Two-dimensional beams are considered here, as in the paper cited above, because of the great simplicity attained when beam and plasma oscillations have the same simple symmetry*.

The beam of the case ii) is ideally constructed by injecting into a plasma a linear beam, modulated at a frequency which is an integral multiple of the gyrofrequency of the beam particles, in a direction which forms a very small angle with a plane perpendicular to $\mathbf{B}_0$.

The plane beam radiates by Čerenkov effect, i.e. only when the ion-velocity is greater than the phase velocity $V = c/n$ of the waves in the plasma; on the contrary, the cylindrical beam can radiate also when this condition is not fulfilled, because of the curvature of the trajectories of the beam particles.

2. Basic equations

Consider now the following linear, nonhomogeneous system of equations [2], [3]:

$$c \operatorname{curl} \mathbf{B} = \dot{\mathbf{E}} + 4\pi (\mathbf{j} + \mathbf{j}_0), \tag{1}$$

$$c \operatorname{curl} \mathbf{E} = -\dot{\mathbf{B}}, \tag{2}$$

$$\varrho_0 \dot{\mathbf{v}} = \frac{1}{c} \mathbf{j} \times \mathbf{B}_0, \tag{3}$$

$$\frac{4\pi}{\omega_p^2} (\dot{\mathbf{j}} + \gamma \mathbf{j}) = \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B}_0 + \frac{1}{e} (m_e - m_i) \dot{\mathbf{v}}, \tag{4}$$

$$\operatorname{div} \mathbf{B} = 0, \tag{5}$$

$$\operatorname{div} \mathbf{E} = 4\pi (\varepsilon + \varepsilon_0); \tag{6}$$

where $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric fields respectively, $\mathbf{j}$ and $\mathbf{j}_0$ are the electric current density of the plasma and the beam; $\mathbf{v}$, ϱ_0, ω_p are the velocity, unperturbed mass density and frequency of the plasma; γ is the mean collision frequency between ions and electrons in the plasma and e is the charge of the proton; m_e and m_i are the electron and ion masses respectively. For simplicity, Eq. (3) does not include the pressure term. Eq. (6), where ε_0 is the electric charge density of the beam, is only a definition of the electric charge density ε of the plasma and will not be used in the following. The solutions of Eqs. (1), (2), (3), (4) and (5) which describe the fields radiated from the beam are obtained from the general integral by assuming "Ausstrahlungsbedingungen", i.e. discarding fields which propagate from infinity towards the beam. In the plane case we chose Cartesian, orthogonal coordinates (x, y, z) such that:

$$\begin{aligned} \mathbf{B} &= B_0 \mathbf{e}_z \\ \mathbf{j} &= \delta(x) \left[I_1 + I e^{i(ly - \omega t)} \right] \mathbf{e}_y, \end{aligned} \tag{7}$$

where $\delta(x)$ is Dirac's function, the constants $I = New$ and $I_1 = N_1 ew$ are the modulated and unmodulated surface current densities of the beam, respectively, and $w = \omega/l$ is the velocity of the particles in the beam; $N_1 \geqslant N$ and N are positive real numbers; ω, l are positive and real.

In the cylindrical case we introduce cylindrical coordinates (r, θ, z) such that:

$$\mathbf{B}_0 = B_0 \mathbf{e}_z.$$

and

$$\mathbf{j}_0 = \delta(r - R) \left(I_1 + I e^{i(m\theta - \omega t)} \right) \mathbf{e}_\theta; \tag{8}$$

where m is a positive integer and $\omega = m\,\omega_g$; $\omega_g = e B_0 / M c$ is the gyrofrequency of the beam particles, whose mass is M; the radius of the beam is $R = w/\omega_g$ and, as in the plane case, $I = New$ and $I_1 = N_1 ew$.

3. The plane beam

In the reference system introduced above we look for solutions of Eqs. (1), (2), (3), (4) and (5) which are independent of z.

If $\mathbf{B}_1$ is a time-independent perturbation of the magnetic field, we obtain

$$\mathbf{B}_1 = \left[-\varepsilon(x) \frac{2\pi}{c} I_1 + \text{const.} \right] \mathbf{e}_z, \tag{9}$$

where $\varepsilon(x)$ is equal to 1 if $x > 0$, and -1 if $x < 0$. We set the constant equal to zero and suppose that $(2\pi/c)\, I_1 \ll B_0$, so that the steady magnetic fields on the left and on the right of the beam are $\sim \mathbf{B}_0$ and we can neglect $\mathbf{B}_1$ in what follows.

Now we look for solutions of Eqs. (1), (2), (3) and (4) of the form

$$\mathbf{B} = \mathbf{B}(x)\, e^{i(ly - \omega t)}, \tag{10}$$

and analogous expressions for $\mathbf{E}$, $\mathbf{j}$ and $\mathbf{v}$. In this way Eq. (5) is identically satisfied and the system Eqs. (1), (2), (3) and (4) separates into two independent parts. The first one contains only the components B_x, B_y, E_z, j_z and v_z and may be reduced to

$$E_z''(x) + (K_E^2 - l^2)\, E_z(x) = 0, \tag{11}$$

where $K_E^2 = (\omega n_E/c)^2$ and $n_E^2 = 1 - [\omega_p^2/(\omega^2 + i\gamma\omega)]$ is the square of the Eccles refractive index. The second part determines B_z, E_x, E_y, j_x, j_y, v_x and v_y and is reduced to

$$B_z''(x) + (K^2 - l^2)\, B_z(x) = -\frac{4\pi}{c} I \left[\delta'(x) - l\alpha\,\delta(x) \right], \tag{12}$$

where $K^2 = (\omega n/c)^2$ is the square refractive index, found by Lüst [4] for plane waves in a plasma with an external magnetic field:

$$n^2 = 1 + \frac{\omega_p^2 (\omega_i \omega_e + \omega_p^2 - \omega^2 - i\gamma\omega)}{(\omega_i \omega_e - \omega^2 - i\gamma\omega)(\omega_i \omega_e + \omega_p^2 - \omega^2 - i\gamma\omega) - \omega^2 (\omega_e - \omega_i)^2}, \tag{13}$$

and

$$\alpha = \frac{\omega\, \omega_p^2 (\omega_e - \omega_i)}{(\omega_i \omega_e - \omega^2 - i\gamma\omega)(\omega_i \omega_e + \omega_p^2 - \omega^2 - i\gamma\omega) - \omega^2 (\omega_e - \omega_i)^2}; \tag{14}$$

Eq. (11) describes perturbations independent of the beam. They are either waves propagating from

* The only linear beam which has simple symmetry properties i.e., a beam parallel to $\mathbf{B}_0$, will be studied in a future paper.

$+\infty$ to $-\infty$ (or reciprocally), if K_E^2 is real and greater than l^2, or monotonic unbounded functions, if K_E^2 is real and positive but smaller than l^2, or a product of such kinds of solutions, if K_E^2 is complex; so we choose the trivial solution $E_z=0$. It then follows:

$$B_x = B_y = j_z = v_z = 0 \, .$$

On the contrary, when we impose $w \geqslant V$ (a restriction analogous to Čerenkov's condition) and the "Ausstrahlungsbedingungen" on Eq. (12), it then supplies the magnetic field radiated from the beam:

$$B_z(x) = \theta(-x)\, b_1 e^{-i\sqrt{K^2-l^2}\,x} + \theta(x)\, b_2 e^{i\sqrt{K^2-l^2}\,x}, \tag{15}$$

where $\theta(x)=1$ if $x>0$, and 0 if $x<0$. The constants b_1 and b_2 satisfy identically the equation:

$$\left(-b_1 e^{-i\sqrt{K^2-l^2}\,x} + b_2 e^{i\sqrt{K^2-l^2}\,x}\right)\delta'(x) + 2i\sqrt{K^2-l^2}\,(b_1+b_2)\,\delta(x) = -\frac{4\pi}{c} I\,[\delta'(x) - l\alpha\delta(x)]\,. \tag{16}$$

Remembering that

$$f(x)\,\delta'(x) = -\delta(x)\,f'(x)\,,$$

provided that

$$f(0) = 0\,,$$

we get

$$-b_1 + b_2 = -\frac{4\pi}{c} I\,, \quad i\sqrt{K^2-l^2}\,(b_1+b_2) = \frac{4\pi}{c} I l\alpha\,,$$

and (17)

$$B_z(x) = -\frac{2\pi}{c}\frac{I}{\sqrt{K^2-l^2}}\left\{-\theta(-x)\left[\sqrt{K^2-l^2} - i l\alpha\right] e^{-i\sqrt{K^2-l^2}\,x} + \theta(x)\left[\sqrt{K^2-l^2} + i l\alpha\right] e^{i\sqrt{K^2-l^2}\,x}\right\}. \tag{18}$$

Then from Eqs. (1), (2), (3), (4) and (18) we obtain Eq. (19), see bottom of page.

We are interested in the time-mean value of the energy flux, given by the real part of the complex Poynting vector [5]

$$\bar{\mathbf{S}} = \frac{c}{8\pi}\,\mathrm{Re}\,(\tilde{\mathbf{E}} \times \tilde{\mathbf{B}}^*)\,, \tag{20}$$

where $\tilde{\mathbf{B}}^*$ is the complex conjugate of $\tilde{\mathbf{B}}$.

In the remainder of this Section we shall consider, for simplicity, a non-dissipative plasma i.e. we will put $\gamma=0$, and we shall study Eq. (20) only in the frequency regions in which n_0^2, defined as

$$n_0^2 = n^2(\gamma=0) = 1 + \frac{\omega_p^2\,(\omega_i\,\omega_e + \omega_p^2 - \omega^2)}{(\omega_i\omega_e - \omega^2)(\omega_i\omega_e + \omega_p^2 - \omega^2) - \omega^2(\omega_e - \omega_i)^2}\,, \tag{21}$$

is >1 in the other regions; there cannot be energy propagation: this fact is obvious for $n_0^2<0$. If, on the contrary, $0<n_0<1$, we still assume that the energy flux is equal to zero due to the fact that the waves are monochromatic. In this way we find:

$$\begin{aligned} \bar{S}_x &= \varepsilon(x)\,\frac{\pi\omega}{2}\left(\frac{I}{cK}\right)^2 \frac{K^2 - l^2(1-\alpha^2)}{\sqrt{K^2-l^2}}\,, \\ \bar{S}_y &= \frac{\pi\omega l}{2}\left(\frac{I}{cK}\right)^2 \frac{K^2 - l^2(1-\alpha^2)}{K^2-l^2}\,, \\ \bar{S}_z &= 0\,. \end{aligned} \tag{22}$$

As a Čerenkov radiation, the vector $\bar{\mathbf{S}}$, as all other quantities, is independent of the mass of the beam particles; but we have to keep in mind that in a magnetic field a beam, carrying a given charge and current, may be considered as a plane beam only when the masses of its particles are very large.

The Čerenkov angle θ between the vectors $\bar{\mathbf{S}}$ and $\mathbf{w}$ is given by

$$\bar{S}_x/\bar{S}_y = \varepsilon(x)\sqrt{K^2/l^2 - 1} = \varepsilon(x)\sqrt{w^2/V^2 - 1} = \tan\theta\,. \tag{23}$$

Hence the energy propagates with the velocity c/n_0 in the two directions which form angles θ and $-\theta$ with $\mathbf{w}$.

$$\begin{aligned}
E_x &= \frac{2\pi I\omega}{(cK)^2\sqrt{K^2-l^2}}\left\{-\theta(-x)\left[l(1-\alpha^2)\sqrt{K^2-l^2} - i\alpha K^2\right]e^{-i\sqrt{K^2-l^2}\,x} + \theta(x)\left[l(1-\alpha^2)\sqrt{K^2-l^2} + i\alpha K^2\right]e^{i\sqrt{K^2-l^2}\,x}\right\}; \\
E_y &= \frac{2\pi I\omega}{(cK)^2\sqrt{K^2-l^2}}\left[l^2(1-\alpha^2) - K^2\right]\left[\theta(-x)\,e^{-i\sqrt{K^2-l^2}\,x} + \theta(x)\,e^{i\sqrt{K^2-l^2}\,x}\right]; \\
j_x &= \frac{-iI}{2n_0^2\sqrt{K^2-l^2}}\left\{\theta(-x)\,(il\alpha - \sqrt{K^2-l^2})\left[l(1-n_0^2) + i\alpha\sqrt{K^2-l^2}\right]e^{-i\sqrt{K^2-l^2}\,x}\right. \\
&\qquad \left. + \theta(x)\,(il\alpha + \sqrt{K^2-l^2})\left[l(1-n_0^2) - i\alpha\sqrt{K^2-l^2}\right]e^{i\sqrt{K^2-l^2}\,x}\right\}; \\
j_y &= \frac{-I}{2n_0^2\sqrt{K^2-l^2}}\left\{\theta(-x)\,(il\alpha - \sqrt{K^2-l^2})\left[l\alpha - i(1-n_0^2)\sqrt{K^2-l^2}\right]e^{-i\sqrt{K^2-l^2}\,x}\right. \\
&\qquad \left. + \theta(x)\,(il\alpha + \sqrt{K^2-l^2})\left[l\alpha + i(1-n_0^2)\sqrt{K^2-l^2}\right]e^{i\sqrt{K^2-l^2}\,x}\right\}; \\
v_x &= \frac{iB_0}{c\varrho_0\omega}\,j_y\,; \\
v_y &= \frac{-iB_0}{c\varrho_0\omega}\,j_x\,.
\end{aligned} \tag{19}$$

In order to discuss other properties of $\overline{S}_x$, it is convenient to use the form:

$$\overline{S}_x = \varepsilon(x)\,\frac{\pi}{2}\left(\frac{Ne}{c}\right)^2 \frac{wV\,(w^2 - V^2(1-\alpha^2))}{\sqrt{w^2 - V^2}}\,. \tag{24}$$

and to express n_0^2 and α by means of the dimensionless parameters

$$\varkappa = \frac{\omega^2}{\omega_\mathrm{i}\,\omega_\mathrm{e}}\,,\quad \sigma = \frac{\omega_\mathrm{p}^2}{\omega_\mathrm{i}\,\omega_\mathrm{e}}\,,\quad \frac{1}{\mu} = \frac{(\omega_\mathrm{e} - \omega_\mathrm{i})^2}{\omega_\mathrm{i}\,\omega_\mathrm{e}} \sim \frac{m_\mathrm{i}}{m_\mathrm{e}}\,. \tag{25}$$

The results are

$$n_0^2 = \frac{\varkappa/\mu - (1-\varkappa+\sigma)^2}{\varkappa/\mu - (1-\varkappa+\sigma)(1-\varkappa)} \tag{26}$$

and

$$\alpha = \frac{-\sigma\sqrt{\varkappa/\mu}}{\varkappa/\mu - (1-\varkappa+\sigma)(1-\varkappa)}\,. \tag{27}$$

It appears from Eq. (24) that in addition to the resonance for $w = V$, which is of the kind found by Kippenhahn and de Vries, S_x has infinities in the resonance frequencies of $n^2{}_0$ (and of α consequently), resonances which have already been studied by K. Körper [6].

When $\sigma \ll 1$, n_0^2 has poles at $\varkappa \sim \mu$ and $\varkappa \sim 1/\mu$; when $\sigma \gg 1/\mu$ the poles are at $\varkappa \sim 1$ and $\varkappa \sim \sigma$. In Körper's terminology the resonances at $\varkappa \sim \mu$ and $\varkappa \sim 1$ are "ion-resonances"; those at $\varkappa \sim 1/\mu$ and $\varkappa \sim \sigma$ are "electron-resonances". The last case, $\varkappa \sim \sigma$, will not be considered because in the immediate neighbourhood of σ there are also the two zeros of n_0^2.

Supposing now $w \gg V$, in the first three cases we have the following simplified expressions for Eq. (24):

a) when $\sigma \ll 1$ and $\dfrac{\sigma\mu}{\mu - \varkappa} \gg 1$,

$$\overline{S}_x \sim \varepsilon(x)\,\frac{\pi}{2}\,(Ne)^2\,c\left(\frac{\sigma\mu}{\mu-\varkappa}\right)^{\frac{1}{2}}; \tag{28}$$

b) when $\sigma \gg 1/\mu$ and $\mu\sigma(1-\varkappa) < 1$,

$$\overline{S}_x \sim \varepsilon(x)\,\frac{\pi}{2}\,(Ne)^2\,c\,\frac{1}{\sigma\mu\,[\sigma(1-\varkappa)]^{1/2}}\,; \tag{29}$$

c) when $\sigma \ll 1$ and $\dfrac{\sigma}{(1/\mu) - \varkappa} \gg 1$,

$$\overline{S}_x \sim \varepsilon(x)\,\frac{\pi}{2}\,(Ne)^2\,c\left(\frac{\sigma}{(1/\mu) - \varkappa}\right)^{\frac{1}{2}}. \tag{30}$$

Note that all these expressions are independent of the velocity of the beam particles.

Outside the resonances of n_0^2 we consider two extreme regions:

a) Very high frequencies, i.e. $\varkappa \gg \mathrm{Max}\,(\sigma; 1/\mu)$: here $\overline{S}_x$ has to be assumed equal to zero, because of the inequality $n_0^2 \sim 1 - (\sigma/\varkappa) < 1$.

b) Very low frequencies, i.e. $\varkappa \ll \mathrm{Min}\,[1; \mu(1+\sigma)]$: here we get

$$\overline{S}_x = \varepsilon(x)\,\frac{\pi}{2}\left(\frac{Ne}{c}\right)^2 \frac{wc}{\sqrt{\sigma+1}}\cdot\frac{w^2 - \dfrac{c^2}{\sigma+1}\left[1 - \dfrac{\varkappa}{\mu}\left(\dfrac{\sigma}{\sigma+1}\right)^2\right]}{\sqrt{w^2 - [c^2/(\sigma+1)]}}\,. \tag{31}$$

In the case $\sigma \gg 1$, i.e. for high density plasma, this expression has already been found in [1]; it gives an extremely small energy loss from the beam far from the resonance $w = V$. For instance, when $\varkappa < 1$ it is $\alpha^2 < 1/\mu$; then S_x becomes:

$$\overline{S}_x < \varepsilon(x)\,\frac{\pi}{2}\left(\frac{Ne}{c}\right)^2 \frac{w^2 V}{\mu}\,; \tag{32}$$

the mean relative energy loss of an ion per cm of path $\delta_y E/E$, given by

$$\frac{\delta_y E}{E} = \frac{S_x}{N_1 \dfrac{M}{2} w^3}\,, \tag{33}$$

is, in the case $N = N_1$, smaller than $\varepsilon(x)\,(\pi/\mu)\,r_\mathrm{i} \times N\,(V/w)$, where $r_\mathrm{i} = e^2/Mc^2$ is the classical ion radius.

For a deuteron beam with $N \sim 10^8\ \mathrm{cm}^{-2}$, $\delta_y E/E$ has an extremely small value: $10^{-4}\ V/w\ \mathrm{cm}^{-1}$.

Unfortunately, measurable radiation cannot be obtained even in the resonance of Eq. (24) at $w = V$, as we shall see in the next Section, because this kind of resonance is destroyed by the curvature experienced by the beam in $\mathbf{B}_0$. Appreciable energy might therefore be emitted only in the resonance of n_0^2. We shall discuss this point in Section 5, including, partially at least, the effect of the finite electric conductivity of the plasma.

4. The curved beam

As in the plane case, we suppose $(2\pi/c)\,I_1 \ll B_0$ and look for solutions of Eqs. (1), (2), (3) and (4) of the form

$$\widetilde{\mathbf{B}} = \mathbf{B}(r)\,e^{\mathrm{i}(m\theta - \omega t)} \tag{34}$$

and analogous expressions for $\mathbf{E}$, $\mathbf{j}$ and $\mathbf{v}$. Then we satisfy Eq. (5) identically and get two systems of equations. The first one determines B_r, B_θ, E_z, j_z and v_z; because it turns out that they are independent of the beam quantities, as in Section 3, we choose the trivial solutions $B_r = B_\theta = E_z = j_z = v_z = 0$. The second system of equations furnishes the components B_z, E_r, E_θ, j_z, j_θ, v_r and v_θ and reduces to the Bessel nonhomogeneous equation

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\,\frac{\mathrm{d}}{\mathrm{d}r}\,B_z(r)\right) + \left(K^2 - \frac{m^2}{r^2}\right)B_z(r) = -\frac{4\pi}{c}\,I\left\{\frac{\mathrm{d}}{\mathrm{d}r}\,\delta(r-R) + \frac{1-m\alpha}{R}\,\delta(r-R)\right\} \tag{35}$$

where $K^2 = (\omega n/c)^2$; n^2 and α are defined by Eq. (13) and Eq. (14).

In order to simplify the calculations and discussions we assume in this Section, as we have done for the plane case, that $n^2 = n^2{}_0 > 1$, i.e. that the plasma is nondissipative and that the energy can propagate.

By using the "Ausstrahlungsbedingungen" and regularity conditions in $r = 0$, we get the solution of Eq. (35):

$$B_z(r) = \theta(R-r)\,b_1 J_m(Kr) + \theta(r-R)\,b_2 H_m^{(1)}(Kr), \tag{36}$$

where $J_m(Kr)$ and $H_m^{(1)}(Kr)$ are, respectively, the Bessel and Hankel functions of the first kind. The constants b_1 and b_2 satisfy the equation:

$$
\begin{aligned}
&[b_2 H_m^{(1)}(Kr) - b_1 J_m(Kr)] \frac{\mathrm{d}}{\mathrm{d}r}\, \delta(r-R) \\
&\quad + \left[2 \frac{\mathrm{d}}{\mathrm{d}r} (b_2 H_m^{(1)}(Kr) - b_1 J_m(Kr)) \right. \\
&\quad \left. + \frac{1}{R} (b_2 H_m^{(1)}(Kr) - b_1 J_m(Kr)) \right] \delta(r-R) \\
&= \frac{-4\pi}{c} I \left[\frac{\mathrm{d}}{\mathrm{d}r} \delta(r-R) + \frac{1-m\alpha}{R}\, \delta(r-R) \right]. \qquad (37)
\end{aligned}
$$

If we proceed as in Section 3, we obtain

$$
\begin{aligned}
B_z(r) = -\frac{2\mathrm{i}\pi^2 I}{c} \{&\theta(R-r)\, h_m(\alpha, KR)\, J_m(Kr) \\
&+ \theta(r-R)\, j_m(\alpha, KR)\, H_m^{(1)}(Kr)\} \qquad (38)
\end{aligned}
$$

where

$$
\begin{aligned}
h_m(\alpha, KR) &= m\alpha H_m^{(1)}(KR) + KR\, H_m^{(1)\prime}(KR) \\
j_m(\alpha, KR) &= m\alpha J_m(KR) + KR\, J_m'(KR)\,.
\end{aligned} \qquad (39)
$$

In the above equations the functions $J_m'(KR)$ and $H_m^{(1)\prime}(KR)$ denote the derivatives with respect to Kr of $J_m(Kr)$ and $H_m^{(1)}(Kr)$, respectively, evaluated at $r=R$. The function $\theta(x)$, defined previously, is unity for $x>0$ and zero for $x<0$.

The other components follow from Eqs. (1), (2), (3), (4) and (38) and are given at the bottom of the page, in Eq. (40).

The components of the real part of the complex Poynting vector (20) are:

$$
\begin{aligned}
\overline{S}_r &= \theta(r-R) \left(\frac{\pi I}{cK}\right)^2 [j_m(\alpha, KR)]^2 \frac{\omega}{r}\,; \\
\overline{S}_\theta &= \frac{\pi\omega}{2} \left(\frac{\pi I}{cK}\right)^2 \Big\{ \theta(R-r)\, |h_m(\alpha\, KR)|^2 \\
&\qquad \times \left[\frac{m}{r} J_m^2(Kr) + \alpha K J_m(Kr) J_m'(Kr) \right] \\
&\quad + \theta(r-R)\, |j_m(\alpha, KR)|^2 \\
&\quad \mathrm{Re} \left[\frac{m}{r} (H_m^{(1)}(Kr))^2 + \alpha K H_m^{(1)}(Kr)\, H_m^{(1)\prime}(Kr) \right] \Big\}\,; \\
S_z &= 0\,. \qquad (41)
\end{aligned}
$$

As we are interested in the radiation close to the beam we consider $\overline{S}_r$ on the external side of the cylinder surface $r=R$:

$$
\overline{S}_r^{>}(R) = \left(\frac{\pi N e V}{c}\right)^2 m w \left[\alpha J_m\left(\frac{mw}{V}\right) + \frac{w}{V} J_m'\left(\frac{mw}{V}\right) \right]^2. \qquad (42)
$$

Now we consider (20) in the neighbourhood of the poles of n_0^2, where $w \gg V$ and:

$$
J_m\left(m \frac{w}{V}\right) \sim \left(\frac{2V}{\pi m w}\right)^{\frac{1}{2}} \cos\left(m \frac{w}{V} - \frac{2m+1}{4}\pi\right). \qquad (43)
$$

By writing n_0^2 and α in terms of $\varkappa$, σ and μ, as in Eqs. (26), (27), we get the following approximate expressions:

a) "Ion-resonance" region: $\sigma \ll 1$, $\sigma\mu/(\mu-\varkappa) \gg 1$

$$
\overline{S}_r^{>}(R) \sim 2\pi (Ne)^2 c \left(\frac{\sigma\mu}{\mu-\varkappa}\right)^{\frac{1}{2}} \cos^2\left[m \frac{w}{c} \left(\frac{\sigma\mu}{\mu-\varkappa}\right)^{\frac{1}{2}} - \frac{2m+1}{4}\pi \right] \qquad (44)
$$

b) "Ion-resonance" region: $\sigma \gg 1/\mu$, $\sigma\mu(1-\varkappa) \ll 1$

$$
\overline{S}_r^{>}(R) \sim 2\pi (Ne)^2 c \frac{1}{\sigma\mu[\sigma(1-\varkappa)]^{\frac{1}{2}}} \cos^2\left[m \frac{w}{c} \left(\frac{\sigma}{1-\varkappa}\right)^{\frac{1}{2}} - \frac{2m+1}{4}\pi \right] \qquad (45)
$$

c) "Electron-resonance" region: $\sigma \ll 1$, $\sigma/[(1/\mu)-\varkappa] \gg 1$

$$
\begin{aligned}
\overline{S}_r^{>}(R) &\sim 2\pi (Ne)^2 c \left(\frac{\sigma}{(1/\mu)-\varkappa}\right)^{\frac{1}{2}} \\
&\times \cos^2\left[m \frac{w}{c} \left(\frac{\sigma}{(1/\mu)-\varkappa}\right)^{\frac{1}{2}} - \frac{2m+1}{4}\pi \right]. \qquad (46)
\end{aligned}
$$

These expressions, as functions of n_0, have the form

$$
c_1 n_0 \cos^2(c_2 n_0 + c_3)\,,
$$

where c_1, c_2 and c_3 are constants. When $n_0 \to \infty$, they have extremely rapid oscillations. Hence it has perhaps more physical meaning to replace them by the mean value in a period 2π:

$$
\frac{c_1}{2\pi} \int_{c_2 n_0}^{c_2 n_0 + 2\pi} n_0' \cos^2(c_2 n_0' + c_3)\, \mathrm{d}(c_2 n_0')\,. \qquad (47)
$$

In this way, if $c_2 n_0 \gg 2\pi$, Eqs. (44)—(46) become:

$$
\overline{S}_r^{>}(R) \sim \pi (Ne)^2 c \left(\frac{\sigma\mu}{\mu-\varkappa}\right)^{\frac{1}{2}}, \qquad (44\mathrm{a})
$$

$$
\overline{S}_r^{>}(R) \sim \pi (Ne)^2 c \frac{1}{\mu\sigma[\sigma(1-\varkappa)]^{\frac{1}{2}}}, \qquad (45\mathrm{a})
$$

$$
\overline{S}_r^{>}(R) \sim \pi (Ne)^2 c \left(\frac{\sigma}{1/\mu - \varkappa}\right). \qquad (46\mathrm{a})
$$

$$
\begin{aligned}
E_r &= \frac{2\mathrm{i}\pi^2 I}{\omega n_0^2} \left\{ \theta(R-r)\, h_m(\alpha, KR) \left[\frac{m}{r} J_m(Kr) + \alpha K J_m'(Kr) \right] + \theta(r-R)\, j_m(\alpha, KR) \left[\frac{m}{r} H_m^{(1)}(Kr) + \alpha K H_m^{(1)\prime}(Kr) \right] \right\}; \\
E_\theta &= -\frac{2\pi^2 I}{\omega n_0^2} \left\{ \theta(R-r)\, h_m(\alpha, KR) \left[\frac{m\alpha}{r} J_m(Kr) + K J'_m(Kr) \right] + \theta(r-R)\, j_m(\alpha, KR) \left[\frac{m\alpha}{r} H_m^{(1)}(Kr) + K H_m^{(1)\prime}(Kr) \right] \right\}; \\
j_r &= \frac{\pi I}{2 n_0^2} \left\{ \theta(R-r)\, h_m(\alpha, KR) \left[\frac{(n_0^2-1)m}{r} J_m(Kr) - \alpha K J'_m(Kr) \right] + \theta(r-R)\, j_m(\alpha, KR) \left[\frac{(n_0^2-1)m}{r} H_m^{(1)}(Kr) - \alpha K H_m^{(1)\prime}(Kr) \right] \right\}; \\
j_\theta &= \frac{-\mathrm{i}\pi I}{2 n_0^2} \left\{ \theta(R-r)\, h_m(\alpha, KR) \left[\frac{m\alpha}{r} J_m(Kr) - K(n_0^2-1) J'_m(Kr) \right] + \theta(r-R)\, j_m(\alpha, KR) \left[\frac{m\alpha}{r} H_m^{(1)}(Kr) - K(n_0^2-1) H_m^{(1)\prime}(Kr) \right] \right\}; \\
v_r &= \frac{\mathrm{i} B_0}{c\omega\varrho_0} j_\theta\,; \qquad v_\theta = \frac{-\mathrm{i} B_0}{c\omega\varrho_0} j_r\,. \qquad (40)
\end{aligned}
$$

In each of these cases, the radiation is just the sum of the radiation emitted from the two sides of the plane beam in the corresponding situations, Eqs. (28)—(30).

With the exception of these cases, when $R < \infty$, $S_r^>(R)$ has no poles and satisfies the inequality:

$$\overline{S}_r^>(R) \leqslant \left(\frac{\pi N e V}{c}\right)^2 \frac{m w}{2} \left(\alpha + \frac{w}{V}\right)^2, \qquad (47)$$

which follows from Eq. (42) by remembering that [7]

$$|J_m(x)| \leqslant \frac{1}{\sqrt{2}} \qquad (m = 1, 2, 3, \ldots).$$

Moreover, it is easy to see directly from Eqs. (42) and (43) that, when $\sigma > 1/\mu$, $w \gg V$ and $\varkappa < 1$, i.e. at low frequencies and high density plasma*, $S_r^>(R)$ has the same extremely small order of magnitude as indicated in Eq. (32). At very high frequencies, i.e. $\varkappa \gg \max(\sigma, 1/\mu)$, $S_r^>(R)$ is to be considered equal to zero because $V > c$ (see last Section).

In concluding this Section we want to verify that Eq. (42) goes into the equivalent equation for the plane case when $R \to \infty$. More exactly, we want to show that the same operation which transforms, in the limit $R \to \infty$, the curved beam into the plane one, transforms S_r into the x-component of Eq. (20). This can be done, provided we use in Eq. (20) a solution of Eq. (12)—where $\delta(x)$ is replaced by $\delta(x - R)$—which doesn't propagate in the region $x < R$, as in the cylindrical case; for instance, this could happen because of a "mirror" situated in an appropriately chosen plane $x = a < R$. Such a solution, valid for $x > a$, is

$$B_z(x) = b_1\left[-e^{i\sqrt{K^2-l^2}(x-a)} + \theta(R-x)\,e^{-i\sqrt{K^2-l^2}(x-a)}\right] + b_2\,\theta(x-R)\,e^{i\sqrt{K^2-l^2}\,x}. \qquad (48)$$

When the constants b_1 and b_2 are determined as in Section 3, we obtain:

$$B_z(x) = \frac{2\pi I}{c\sqrt{K^2-l^2}} \Big\{ (\sqrt{K^2-l^2} - i l \alpha)\, e^{i\sqrt{K^2-l^2}(R-a)} \left[-e^{i\sqrt{K^2-l^2}(x-a)} + \theta(R-x)\, e^{-i\sqrt{K^2-l^2}(x-a)}\right] + (\sqrt{K^2-l^2} + i l \alpha)\, e^{-i\sqrt{K^2-l^2}\,R}\,\theta(x-R)\, e^{i\sqrt{K^2-l^2}\,x} \Big\}. \qquad (49)$$

For the x-component of Eq. (20), calculated on the side of the beam looking towards the positive x-direction, we obtain the expression**:

$$S_x^>(R) = 2\pi\left(\frac{Ne}{c}\right)^2 \frac{V w}{\sqrt{w^2 - V^2}} \times \left\{\sqrt{w^2-V^2}\cos\left[\frac{l}{V}\sqrt{w^2-V^2}(R-a)\right] + \alpha V \sin\left[\frac{l}{V}\sqrt{w^2-V^2}(R-a)\right]\right\}^2 \qquad (50)$$

Now in order to transform the curved beam into the plane one, we put:

$$R = m/l; \quad m \to \infty; \quad w_{\text{cylinder}} = w_{\text{plane}} = \text{constant}.$$

Then, in the case $w = V$, we note that [7] the functions $J_m(m)$ and $J_m'(m)$ are positive decreasing functions of m and are given (for a sufficiently large m) by the Cauchy formulae:

$$J_m(m) \sim \frac{\Gamma(^1/_3)\, m^{-\frac{1}{3}}}{2^{\frac{2}{3}} \cdot 3^{\frac{1}{6}}\, \pi}; \quad J_m'(m) \sim \frac{3^{\frac{1}{6}} \cdot \Gamma(^2/_3)\, m^{-\frac{2}{3}}}{2^{\frac{1}{3}}\, \pi}. \qquad (51)$$

We then get from Eq. (42)

$$\overline{S}_r^>(R) \sim 0{,}2 \left(\frac{\pi N e \alpha}{c}\right)^2 m^{\frac{1}{3}}\, V^3. \qquad (52)$$

Hence, in the limit, $S_r^>(R)$ has the resonance at $w = V$ as in Eq. (50). However, in order to see how effective such a resonance is, let us consider the mean relative energy loss of a deuteron per cm of path, given by Eq. (33). For instance, in the case $\sigma \gg 1$ and $\varkappa$ sufficiently smaller than 1, so that $\alpha < 1/\mu$, the mean relative energy loss is

$$\frac{\delta_\theta E}{E} \sim 1{,}2\, \pi N r_i\, \alpha^2 m^{\frac{1}{3}} < \frac{\pi N r_i}{\mu}. \qquad (53)$$

For a beam with $N = 10^8$ deuterons per cm^2, the value of this expression is $< 10^{-4}$ cm^{-1}.

Hence, the resonance found in the plane case for $w = V$ is practically ineffective. Measurable radiation can then only be emitted in the resonances of n_0^2, as we shall see in the next Section, when the finite conductivity of the plasma is taken into account.

In the case $w < V$, we substitute in Eq. (42) the inequality [7]

$$J_m\left(\frac{m w}{V}\right) < m^{-\frac{1}{2}} \left[f\left(\frac{w}{V}\right)\right]^m, \quad \text{where } \left|f\left(\frac{w}{V}\right)\right| < 1, \qquad (54)$$

which follows from Carlini's formula. We then have, according to the Čerenkov condition,

$$\lim_{R \to \infty} \overline{S}_r^>(R) = 0. \qquad (55)$$

On the contrary, if $w > V$, we introduce in Eq. (42) the Langer uniform expansion [8]:

$$J_m\left(\frac{m w}{V}\right) = \left(1 - \frac{1}{W \tan W}\right)^{\frac{1}{2}} \left[J_{\frac{1}{3}}(z)\cos\frac{\pi}{6} - Y_{\frac{1}{3}}(z)\sin\frac{\pi}{6}\right] + O\left(m^{-\frac{4}{3}}\right), \qquad (56)$$

where

$$W = \left[\left(\frac{w}{V}\right)^2 - 1\right]^{\frac{1}{2}}, \quad z = m\,(W - \tan^{-1} W). \qquad (57)$$

* In these conditions a density-modulated beam of electrons cannot be realized and in order to have a deuteron beam we must assume m of the order of unity.

** Note that in the resonance regions of n_0^2, when $w \gg V$, Eq. (50) takes the same form as Eqs. (44), (45) and (46) and that, when we calculate the mean value of these expressions as we did in Section 4, we again find the radiation equal to the sum of the radiation emitted from the two sides of the beam of Section 3 in the corresponding situations.

If $z \gg 1$, $J_{\frac{1}{3}}(z)$ is given by Eq. (43) and $Y_{\frac{1}{3}}(z) \sim \sim (2/\pi z)^{\frac{1}{2}} \sin[z-(5\pi/12)]$. Then, by equating the so-obtained expression from Eq. (42) with Eq. (50), we have an equation for the mirror position a.

5. On the peak value of S

When the finite electric conductivity of the plasma is accounted for, the expressions for S, given in Sections 3 and 4, are no longer appropriate. The squared refractive index n^2 is a bounded complex function [6] whose real part in the resonance regions differs very much from n_0^2; moreover, the imaginary part of n indicates energy is absorbed by the plasma.

It is not our intention to study the problem in its most general form. In what follows we shall restrict ourselves, for simplicity, to the evaluation of the peak value of the radiation from the plane and from the curved beam at the "ion-resonance" frequency. This is practically the only resonance frequency of interest for experiments. Finally, the resonance breadth and the attenuation distance of the waves will be briefly examined.

Consider now the general forms of n^2 and α given by Eqs. (13) and (14), respectively, and express them in terms of the dimensionless parameters $\varkappa$, σ and μ introduced in Eq. (25) and of the new parameter $\tau = \gamma/\sqrt{\omega_i \omega_e}$. By supposing $\tau \ll 1$ and denoting with $\varkappa_r$ the value of $\varkappa$ which corresponds to the ion-resonance frequency in absence of collisions, it is easy to find that Re (n^2) and Re (α) reach their peak values at $\varkappa = \varkappa_r - O(\tau)$, where $O(\tau)$ is positive and of the order of τ.

In the following we shall choose, in order to simplify the calculations, just the value $\varkappa = \varkappa_r - O(\tau) + O(\tau^2)$. At this value we have (see Fig. 3)

$$\begin{aligned} \mathrm{Re}\,(n^2) &\equiv \mathrm{Im}\,(n^2) = \bar{n}^2, \\ \mathrm{Re}\,(\alpha) &\equiv \mathrm{Im}\,(\alpha) = \bar{\alpha}, \end{aligned} \tag{58}$$

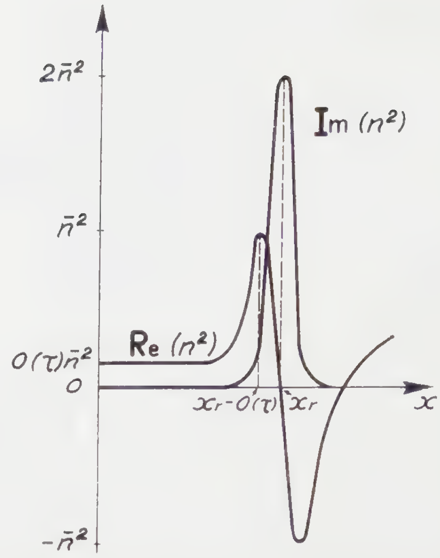

Fig. 3 Schematic behaviour of n^2 in the neighbourhood of $\varkappa_r$. The behaviour of α can be obtained by substituting $\bar{\alpha}$ for $\bar{n}^2$ in the ordinate.

so that we can write

$$\begin{aligned} n^2 &= \bar{n}^2 \sqrt{2}\, e^{i\pi/4} \\ \alpha &= \bar{\alpha} \sqrt{2}\, e^{i\pi/4}. \end{aligned} \tag{59}$$

$\varkappa_r - O(\tau)$, $\bar{n}$ and $\bar{\alpha}$ are real functions of σ; approximate expressions are given in Table I. It follows from Table I that $\bar{\alpha} \gg \bar{n}$.

With these assumptions, the x-component of the complex Poynting vector, Eq. (20), for the plane beam,

$$\bar{S}_x = \frac{c}{8\pi} \mathrm{Re}\left\{\left(\frac{-ic}{\omega n^2}\right)(l\,\alpha\,|B_z|^2 + B_z^* \bar{\mathrm{d}}_x B_z)\right\}, \tag{60}$$

where $\bar{\mathrm{d}}_x$ is the derivative with respect to x, acting only on the continous parts of B_z, reduces to

$$\frac{c}{8\pi\omega} \mathrm{Im}\left\{\frac{B_z^* \bar{\mathrm{d}}_x B_z}{n^2}\right\}. \tag{61}$$

TABLE I.

	$\sigma \ll 1/\mu$	$\sigma \gg 1$
$\varkappa_r - O(\tau)$	$\mu\left\{\sigma+1-\tau(\sigma+2)\left[\mu(\sigma+1)\right]^{\frac{1}{2}}\right\}$	$\frac{\sigma}{\sigma+(1/\mu)}\left[1-\tau\left(\frac{\sigma}{\sigma+(1/\mu)}\right)^{\frac{1}{2}}\right]$
$1/\bar{n}$	$\left[\frac{2\tau(\sigma+2)\mu^{\frac{1}{2}}}{\sigma(\sigma+1)^{\frac{1}{2}}}\right]^{\frac{1}{2}}$	$\frac{(2\tau)^{\frac{1}{2}}}{\{\sigma[\sigma+(1/\mu)]\}^{\frac{1}{4}}}$
$\bar{\alpha}$	$\frac{\sigma}{2\tau\mu^{\frac{1}{2}}(\sigma+2)}$	$\frac{1}{2\tau\mu^{\frac{1}{2}}}$

By using Eqs. (18), (59) and (61) and supposing moreover $w \gg V$, the x-component of the complex Poynting vector, calculated on the side of the beam looking towards the positive x-direction, becomes:

$$\overline{S}_x^{>} = \frac{\pi}{2}(Ne)^2 c \frac{\overline{\alpha}^2}{\overline{n}^3} \cdot \frac{\cos(\pi/8) + \sin(\pi/8)}{2^{1/4}} . \tag{62}$$

On the other side of the beam it is $S_x^{<} = -S_x^{>}$.

In the same way, the radial component of $\overline{\mathbf{S}}$, for the curved beam, takes the form (61) where $\overline{\mathrm{d}}_x$ is replaced by $\overline{\mathrm{d}}_r$. Remembering now that [8]

$$[H_m^{(1)}(KR)]^* = H_m^{(2)}(K^*R) \tag{63}$$

and that for $|KR| \gg m$ the Hankel functions become [8]:

$$H_m^{(1)}(KR) \sim \left(\frac{2}{\pi KR}\right)^{\frac{1}{2}} \exp\left[\mathrm{i}\left(KR - \frac{2m+1}{4}\pi\right)\right]$$

$$H_m^{(2)}(K^*R) \sim \left(\frac{2}{\pi K^*R}\right)^{\frac{1}{2}} \exp\left[-\mathrm{i}\left(K^*R - \frac{2m+1}{4}\pi\right)\right] \tag{64}$$

while $J_m(KR)$ is given by Eq. (43), from Eqs. (38), (59) and (61) we get the following expressions for $\overline{S}_r$ on the two sides of the beam,

$$\overline{S}_r^{>} = -\overline{S}_r^{<} = \frac{\pi}{2}(Ne)^2 c \frac{\overline{\alpha}^2}{\overline{n}^3} \frac{\cos(\pi/8) + \sin(\pi/8)}{2^{1/4}} . \tag{65}$$

For simplicity, we indicate with S the common value of $\overline{S}_x^{>}$ and $\overline{S}_r^{>}$ (R). Note here that the mean energy flux, Eq. (62) or (65), does not depend on the velocity of the beam particles and on the beam curvature. This means that, when the interaction between beam and plasma becomes very strong for a particular value of the modulation frequency, the large modifications induced by the beam in the surrounding plasma react on the field radiated by the beam in such a way that the resulting field comes out to be essentially determined by the plasma properties and no longer by the beam dynamics*. The dependence of Eq. (65) on the plasma properties is better discussed by expressing $\overline{\alpha}^2/\overline{n}^3$ in terms of the magnetic field B_0 (measured in Gauss), the particle density of the plasma d and the plasma temperature T (measured in°K). By using for γ the expression given by SPITZER** [3] we get Table II.

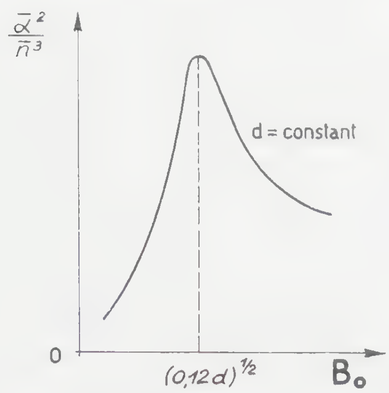

Fig. 4 Schematic behaviour of $\overline{\alpha}^2/\overline{n}^3$ as a function of B_0.

* Something similar happens in the problem of the energy loss of a fast particle traversing matter, see e. g. E. FERMI [9]: here, the interaction between particle and the medium becomes very strong when the particle velocity v approaches c; but then the radiation doesn't increase indefinitely; on the contrary, for velocity larger than a certain v, it takes a value which is independent of v and is determined by the dielectric properties of the medium; such a value diminishes when the density of the medium increases. Note in this connection that our expression, Eq. (24), for instance for a non-dissipative plasma and at low frequencies becomes, when $w \to c$, $\propto B_0/\sqrt{d}$, where d is the particle density of the plasma.

** Remembering that in our cases the current is perpendicular to $\mathbf{B}$, we have to choose the γ used for the transverse conductivity.

TABLE II.

	$d \ll 10^5 \cdot B_0^2$	$d \gg 26 \cdot B_0^2$
$\omega_r = (\omega_i \omega_e \varkappa_r)^{\frac{1}{2}}$ sec^{-1}	$4{,}8 \cdot 10^3 \cdot (B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)^{\frac{1}{2}}$	$8{,}7 \cdot 10^2 \cdot \dfrac{B_0 d^{\frac{1}{2}}}{(B_0^2 + 10^{-5} d)^{\frac{1}{2}}}$
$\dfrac{\overline{\alpha}^2}{\overline{n}^3}$	$\dfrac{10^2 \cdot T^{\frac{3}{4}} B_0^2}{\left[\ln\Lambda \cdot (2B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)(B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)^{\frac{3}{2}}\right]^{\frac{1}{2}}}$	$\dfrac{2 \cdot 10^4 \cdot B_0^{\frac{7}{2}} T^{\frac{3}{4}}}{\left[\ln\Lambda \cdot d^{\frac{5}{2}} (B_0^2 + 10^{-5} \cdot d)^{\frac{3}{2}}\right]^{\frac{1}{2}}}$
$\dfrac{\Delta\omega}{\omega_r}$	$1{,}3 \cdot 10^{-3} \cdot \left[\dfrac{\ln\Lambda \cdot d\,(2B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)}{T^{\frac{3}{2}} B_0^2 (B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)^{\frac{1}{2}}}\right]^{\frac{1}{2}}$	$5{,}5 \cdot 10^{-4} \cdot \left[\dfrac{\ln\Lambda \cdot d^{\frac{3}{2}}}{T^{\frac{3}{2}} B_0 (B_0^2 + 10^{-5} \cdot d)^{\frac{1}{2}}}\right]^{\frac{1}{2}}$
$L = \dfrac{c}{\omega_r \overline{n}}$ cm	$5{,}6 \cdot 10^4 \cdot \left[\dfrac{\ln\Lambda \cdot (2B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)}{T^{\frac{3}{2}} (B_0^2 + 3{,}8 \cdot 10^{-2} \cdot d)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$	$1{,}4 \cdot 10^5 \left[\dfrac{\ln\Lambda \cdot (B_0^2 + 10^{-5} \cdot d)^{\frac{1}{2}}}{T^{\frac{3}{2}} \cdot d^{\frac{1}{2}} B_0}\right]^{\frac{1}{2}}$

Ion-electron collisions tend to destroy the resonance condition; hence it is clear that the mean energy flux must increase with the temperature. Then, from Fig. 4 where the behaviour of $\bar{\alpha}^2/\bar{n}^3$ taken as a function of B_0 is sketched, we see that the peak value S becomes particularly high when the plasma frequency ω_p is of the order of $\sqrt{\omega_i\omega_e}$. From Fig. 5 on the contrary it

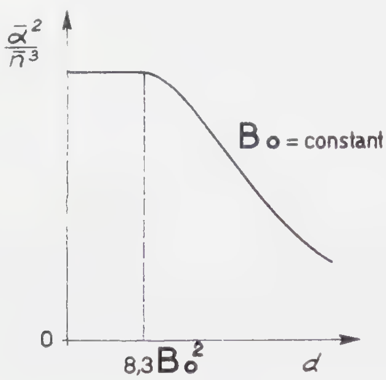

Fig. 5 Schematic behaviour of $\bar{\alpha}^2/\bar{n}^3$ as a function of d.

follows that the plasma has also a kind of screening effect on the beam radiation which increases with the ratio $\omega_p^2/\omega_i\omega_e \propto d/B_0^2$. The greatest value of $\bar{S}$ occurs when $d \sim 8{,}3\ B_0^2$:

$$\bar{S} \sim 0{,}6\cdot 10^2\cdot\frac{\pi}{2}\,(Ne)^2\,\frac{c\,T^{\frac{3}{4}}}{(\ln\Lambda)^{\frac{1}{2}}\,B_0^{\frac{1}{2}}}\ \mathrm{erg\,cm^{-2}\,sec^{-1}}. \tag{66}$$

In this case the mean relative energy loss of a deuteron per cm of path, for the plane beam as well as for the curved one, has the value:

$$\frac{\delta E}{E} \sim 1{,}5\cdot 10^{-14}\,\frac{N\,T^{\frac{3}{4}}}{(\ln\Lambda)^{\frac{1}{2}}\,B_0^{\frac{1}{2}}}\left(\frac{c}{w}\right)^3 \mathrm{cm^{-1}}. \tag{67}$$

In many cases of interest for experiments, however, $\sigma \gtrsim 1/\mu$; then from Table II we can write

$$\bar{S} \sim \left(\frac{N}{d}\right)^2 \frac{B_0^{\frac{7}{2}}\,T^{\frac{3}{4}}}{(\ln\Lambda)^{\frac{1}{2}}}\ \mathrm{erg\,cm^{-2}\,sec^{-1}}, \tag{68}$$

and, for deuteron beams,

$$\frac{\delta E}{E} \sim 0{,}6\cdot 10^{24}\,\frac{N\,B_0^{\frac{7}{2}}\,T^{\frac{3}{4}}}{(\ln\Lambda)^{\frac{1}{2}}\,w^3\,d^2}\ \mathrm{cm^{-1}}. \tag{69}$$

When $B_0 = 10^4$ Gauss, $N = 10^8\ \mathrm{cm^{-2}}$, $d = 10^{13}\ \mathrm{cm^{-3}}$ and $T = 10^6\ {}^\circ\mathrm{K}$, Eq. (69) becomes

$$\frac{\delta E}{E} \sim 2\cdot 10^{-8}\left(\frac{c}{w}\right)^3 \mathrm{cm^{-1}}. \tag{70}$$

The particle velocity w is, in this case, $\sim 0{,}8\cdot 10^7\,\lambda$ and the beam radius $R \sim \lambda/2\pi$, where λ is the wavelength of the modulation.

If λ is of the order of 10 cm, so that $R \sim 1$ or 2 cm and $w \sim 8\cdot 10^7\ \mathrm{cm\,sec^{-1}}$, the value of Eq. (70) is ~ 1.

Of course, the practical interest of the previous equations depends essentially on the value of the "breadth" of the energy release, taken as a function of the modulation frequency. This breadth can be evaluated by looking at the form of n^2 and α. For a displacement $\Delta\varkappa \sim -O(\tau)$ away from the resonance, Re (n^2) and Re (α) become $\sim O(\tau)$-times smaller than $\bar{n}^2$ and $\bar{\alpha}$, respectively, while they go at once to zero for a displacment $\Delta\varkappa \sim O(\tau)$; moreover, except in the neighbourhood of $\varkappa_r$, it is always Im $(n^2) \ll$ Re (n^2) and Im $(\alpha) \ll$ Re (α). Consequently, the value of $\bar{S}$ (and of $\delta_\theta E/E$) becomes negligible compared to $\bar{S}$ (and to $\delta E/E$) for displacements $\Delta\omega$ of the modulation frequency away from the resonance such that:

$$\frac{|\Delta\omega|}{\omega} \sim \left(\frac{O(\tau)}{\varkappa_r}\right)^{\frac{1}{2}}. \tag{71}$$

Approximated expressions of Eq. (71) are given in Table II. In cases where Eq. (69) holds, expression (71) takes the form: $\sim 1\cdot 10^{-3}\,[d\ln\Lambda/(T^{\frac{3}{2}}\,B_0)]^{\frac{1}{2}}$; when Eq. (70) holds, it is $|\Delta\omega|/\omega \sim 3{,}7\cdot 10^{-3}$.

In concluding we note that the energy released by the beam is absorbed by the plasma over a distance, whose order of magnitude L is given by [3]

$$\frac{1}{L} = \mathrm{Im}\left(\frac{\omega\,n}{c}\right). \tag{72}$$

For the explicit form of L, see Table II. When Eq. (70) holds, L is about 10^{-2} cm. L increases with B_0, while it diminishes when T increases, because of the fact that the collisions tend to destroy the resonance and thus deepen the penetration. (The wave attenuation due to the collisions is negligible compared to that due to the absorption at the resonance.)

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THEORY OF ČERENKOV AND CYCLOTRON RADIATIONS IN PLASMAS

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Radiation from a charge q moving in a helix in magnetoplasmas is investigated theoretically. When its speed v is much larger than the thermal velocity, $(m^{-1}\,\mathrm{k}T)^{1/2}$, of the plasma electrons and the gyration frequency is much smaller than the plasma frequency ω_0, the radiation power from the charge is $(q^2\,\omega_0^2/2v)\ln(v^2/m^{-1}\,\mathrm{k}T)$. Cyclotron radiation from an electron with non-relativistic speed decreases to zero as plasma density increases. For a positron in a dilute plasma, however, the radiation is strengthened. This strengthened radiation from a positron again decreases with increasing ω_0^2/ω_H^2 and becomes zero for $\omega_0^2/\omega_H^2 \geq 2$ (ω_H = gyration frequency of the plasma electron). Damping of the Cerenkov radiation due to collisions of plasma electrons is also discussed.

1. Introduction

In this paper we investigate theoretically the radiation from a fast moving electron in a uniform infinite plasma. The interactions of the electron with the plasma can be classified into two parts: the microscopic one (close and distant encounters) and the macroscopic one (resonances with the plasma as a continuous medium).

The distant encounter is characterized by the impact parameter which lies between e^2/mv^2 and the Debye length l_D. Here $-e$, m and v are the charge, mass and velocity of the moving electron. The impact parameter plays an essential role in such irreversible processes as temperature relaxation and electric conduction. A fast moving electron loses its kinetic energy mostly due to the distant encounter with plasma electrons. The time rate of energy loss of this type is given by

$$\frac{e^2\omega_0^2}{v}\ln\frac{mv^2 l_D}{e^2}, \tag{1}$$

where ω_0 is the plasma frequency.

The close encounter with ions causes the bremsstrahlung.

The energy loss due to the Čerenkov radiation from the moving electron and the shielding effect on the cyclotron radiation are two examples of resonant interaction characterized by the macroscopic electromagnetic field of wavelength longer than the Debye length.

A charged particle moving uniformly along the magnetic field in a plasma radiates electromagnetic waves [1, 2, 3], called the Čerenkov radiation [4, 5, 6]. A charged particle gyrating in a magnetic field in vacuum radiates electromagnetic waves, the frequencies being close to the gyration frequency and its higher harmonics [7, 8, 9]. This is the cyclotron radiation. For a charged particle moving helically in a magnetoplasma these two types of radiation can not in general be treated separately, although for higher harmonics of the cyclotron radiation the surrounding medium can be approximated by vacuum [10]. Theories of radiation in a magneto-plasma are mostly formal and give few concrete results [3, 10].

A purpose of the present article is to show that under certain conditions we can treat these two radiations separately and thus obtain useful theorems. We use the Fourier-expansion method developed by SITENKO and KOLOMENSKII [12].

2. Preliminaries

Starting from the Maxwell equations

$$\begin{aligned} \mathrm{rot}\,\mathbf{E}(\mathbf{r},t) &= -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{H}(\mathbf{r},t)\,, \\ \mathrm{rot}\,\mathbf{H}(\mathbf{r},t) &= \frac{1}{c}\frac{\partial}{\partial t}\mathbf{D}(\mathbf{r},t) + \frac{4\pi}{c}\mathbf{J}(\mathbf{r},t)\,, \end{aligned} \tag{2a}$$

we make the Fourier expansion for each quantity, in such a way as

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= \iint \mathbf{E}(\mathbf{k},\omega)\exp(\mathrm{i}\,\mathbf{k}\cdot\mathbf{r}-\mathrm{i}\,\omega t)\,\mathrm{d}\mathbf{k}\,\mathrm{d}\omega\,, \\ \mathbf{E}(-\mathbf{k},-\omega) &= \mathbf{E}^*(\mathbf{k},\omega)\,,\ \text{etc}. \end{aligned} \tag{2b}$$

Then for the Fourier components the following equations hold:

$$\mathrm{i}\,\mathbf{k}\times\mathbf{E} = \frac{\mathrm{i}\,\omega}{c}\mathbf{H}\,,\quad \mathrm{i}\,\mathbf{k}\times\mathbf{H} = \frac{-\mathrm{i}\,\omega}{c}\mathbf{D} + \frac{4\pi}{c}\mathbf{J}\,. \tag{2c}$$

On eliminating $\mathbf{H}$, we have

$$\mathbf{D} + \frac{c^2}{\omega^2}(\mathbf{k}\mathbf{k}\cdot\mathbf{E} - k^2\mathbf{E}) = -\mathrm{i}\frac{4\pi}{\omega}\mathbf{J}\,. \tag{3}$$

Using the connection between the Fourier components of the electric displacement and the electric field intensity

$$\begin{aligned} \mathbf{D}(\mathbf{k},\omega) &= \boldsymbol{\varepsilon}(\omega)\cdot\mathbf{E}(\mathbf{k},\omega)\,, \\ \varepsilon_{ik}(-\omega) &= \varepsilon_{ik}^*(\omega)\,, \end{aligned} \tag{4}$$

we obtain

$$\mathsf{T}\cdot\mathbf{E} = -\mathrm{i}\frac{4\pi}{\omega}\mathbf{J}\,, \tag{5}$$

where

$$T_{ik} = n^2 (\varkappa_i \varkappa_k - \delta_{ik}) + \varepsilon_{ik}\,, \quad T_{ik}(-\omega) = T^*_{ik}(\omega)\,, \quad (6)$$

$$n = \frac{kc}{|\omega|}\,, \quad \mathbf{k} = \frac{n\,\omega}{c}\,\boldsymbol{\varkappa}\,. \quad (7)$$

By use of the inverse tensor T^{-1} the solution of Eq. (5) can be given in the form

$$\mathbf{E} = -\mathrm{i}\,\frac{4\pi}{\omega}\,\mathsf{T}^{-1}\cdot\mathbf{J}\,. \quad (8)$$

We have therefore

$$\mathbf{E}(\mathbf{r},t) = -\mathrm{i}4\pi\int\!\!\int \mathsf{T}^{-1}\cdot\mathbf{J}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\,\omega t)\,\mathrm{d}\mathbf{k}\,\omega^{-1}\,\mathrm{d}\omega$$

and hence (9)

$$\mathbf{H}(\mathbf{r},t) = -\mathrm{i}4\pi\int\!\!\int n\boldsymbol{\varkappa}\times\mathsf{T}^{-1}\cdot\mathbf{J}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t)\,\mathrm{d}\mathbf{k}\omega^{-1}\mathrm{d}\omega\,. \quad (10)$$

The integration variable can be transformed according to

$$\mathrm{d}\mathbf{k}\,\omega^{-1}\,\mathrm{d}\omega = c^{-3}\,\mathrm{d}\Omega\,n^2\mathrm{d}n\,|\omega|\,\omega\,\mathrm{d}\omega \quad (11)$$

where $\mathrm{d}\,\Omega$ is the solid angle element of the direction of the wave vector $\mathbf{k}$.

We consider the electromagnetic field produced in the medium in which a point charge q is moving with a velocity $\mathbf{v}(t)$. Denoting by $\mathbf{r}_e(t)$ the position of the charged particle at time t, the current is represented in the form $q\mathbf{v}\,\delta(\mathbf{r}-\mathbf{r}_e)$ with the Dirac δ-function. Then, its Fourier component is calculated to be

$$\mathbf{J}(\mathbf{k},\omega) = \frac{q}{(2\pi)^4}\int \mathbf{v}\exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{r}_e + \mathrm{i}\omega t)\,\mathrm{d}t\,. \quad (12)$$

The energy radiated from the charge is given by a surface integral of the Poynting vector $\mathbf{S} = (4\pi)^{-1}c\,(\mathbf{E}\times\mathbf{H})$. The Poynting vector, however, is complicated in general, and the calculation is performed only in particular cases (see Section 5).

On the other hand, the time rate of energy loss by the moving charge, $-\mathrm{d}W/\mathrm{d}t$, due to interactions with the surrounding continuous medium can be calculated by use of the relationship

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = -q\,\mathbf{v}\cdot\mathbf{E}(\mathbf{r}_e,t)$$
$$= 4\pi q\,\mathrm{i}\int\!\!\int \mathbf{v}\cdot\mathsf{T}^{-1}\cdot\mathbf{J}\exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r}_e - \mathrm{i}\omega t)\,\mathrm{d}\mathbf{k}\omega^{-1}\mathrm{d}\omega\,, \quad (13)$$

in which the field $\mathbf{E}$ is taken at the point where the charge is located. This energy loss is not always equal to the energy radiated from the charge even if the electric resistivity of the medium is negligible. The charged particle, in some cases, does work on the medium in the neighbourhood of its track without radiating energy. There are two criteria in this respect.

The condition that absorption is absent is that the tensor $\boldsymbol{\varepsilon}$ is hermitian.

$$\varepsilon_{ik}(\omega) = \varepsilon^*_{ki}(\omega)\,. \quad (14)$$

The time rate of radiation from a moving charge is equal to the time rate of energy loss given by Eq. (13) when the dielectric tensor $\boldsymbol{\varepsilon}(\omega)$ is hermitian in the domain of ω which contributes to the integral of Eq. (13).

An example of energy loss without radiation is given in the next Section.

Plane electromagnetic waves which are propagated in a source-free magneto-plasma are given by Eq. (5) with $\mathbf{J}=0$:

$$\mathsf{T}\cdot\mathbf{E} = 0\,, \quad (5a)$$

the index of refraction, n, being given by the relationship

$$\det\mathsf{T} = 0\,. \quad (15)$$

There exist for n^2 two finite roots $n_1{}^2$ and $n_2{}^2$. (These roots are real when the tensor $\boldsymbol{\varepsilon}(\omega)$ is hermitian.)

When these roots are real and when the integral with respect to n^2 is determined by the integrand at a positive root or roots, then the energy loss is entirely due to the radiation with these indexes of refraction.

When, in particular, the particle velocity $\mathbf{v}$ is a constant, the Fourier component of the electric current, Eq. (12), reduces to

$$\mathbf{J} = \frac{q\mathbf{v}}{(2\pi)^4}\int\exp(-\mathrm{i}\mathbf{k}\cdot\mathbf{v}t + \mathrm{i}\omega t)\,\mathrm{d}t$$
$$= \frac{q\mathbf{v}}{(2\pi)^3}\,\delta(\mathbf{k}\cdot\mathbf{v} - \omega)$$
$$= \frac{q\mathbf{v}}{(2\pi)^3}\,\frac{1}{|\omega|}\,\delta\!\left(\frac{n}{c}\,\boldsymbol{\varkappa}\cdot\mathbf{v} - 1\right), \quad (16)$$

the particle position at $t=0$ being chosen as the coordinate origin. Then, the time rate of energy loss, Eq. (13), becomes

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{q^2}{2\pi^2c^3}\,\mathrm{i}\int\!\!\int\!\!\int \mathbf{v}\cdot\mathsf{T}^{-1}\cdot\mathbf{v}\,\delta\!\left(\frac{n}{c}\boldsymbol{\varkappa}\cdot\mathbf{v} - 1\right)\mathrm{d}\Omega n^2\mathrm{d}n\omega\mathrm{d}\omega\,, \quad (17)$$

where Eq. (11) has been used. This expression was first given by SITENKO and KOLOMENSKII [12].

3. The polarization loss

As a preparation we first consider a collisionless plasma with no magnetic field in which a charged particle is moving. We assume that the speed v of the charged particle is much larger than the thermal velocity of the plasma electron,

$$\ln(v^2/m^{-1}\mathrm{k}\,T) \gg 1\,, \quad (18)$$

where k is the Boltzmann constant, T is the plasma temperature, and m is the electron mass.

For a scalar dielectric constant ε, we have

$$\mathsf{T}^{-1}\cdot\mathbf{v} = \frac{v}{\varepsilon(n^2-\varepsilon)}\,(n^2\varkappa_x\varkappa_z,\, n^2\varkappa_y\varkappa_z,\, n^2\varkappa_z{}^2 - \varepsilon)\,, \quad (19)$$

the z-axis being taken along the velocity $\mathbf{v}$. The energy loss, Eq. (17), thus takes the simple form

$$-\frac{\mathrm{d}W}{\mathrm{d}t} =$$
$$\frac{q^2v^2}{\pi c^3}\,\mathrm{i}\int\!\!\int\!\!\int\frac{n^2\cos^2\theta - \varepsilon}{\varepsilon(n^2-\varepsilon)}\,\delta(n\beta\cos\theta - 1)\sin\theta\mathrm{d}\theta\,n^2\mathrm{d}n\omega\,\mathrm{d}\omega \quad (20\mathrm{a})$$

in which $\beta \equiv v/c$ and θ is the angle between $\mathbf{v}$ and $\boldsymbol{\varkappa}$. Hence

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{q^2}{\pi v}\,\mathrm{i}\iint\limits_{n>\beta^{-1}} \frac{1-\beta^2\varepsilon}{\varepsilon\,(n^2-\varepsilon)}\, n\,\mathrm{d}n\,\omega\,\mathrm{d}\omega\,. \qquad (20\,\mathrm{b})$$

The plasma can be treated as a continuous medium with the dielectric constant

$$\varepsilon = 1 - (\omega_0{}^2/\omega^2) \qquad (21\,\mathrm{a})$$

for wavelengths longer than the Debye length l_{D}, namely for

$$\beta n < v/(|\omega|\, l_{\mathrm{D}})\,. \qquad (21\,\mathrm{b})$$

The limit $v/(|\omega|\, l_{\mathrm{D}})$ is a large number by virtue of our assumption, and the integral with respect to βn can be cut off at this limit. The integral of Eq. (20b) is determined by the values of the integrand at its poles $\omega = \pm\omega_0$ and we have

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{q^2\,\omega_0{}^2}{v}\int\limits_{1/\beta}^{v/\omega_0 l_D}\frac{\mathrm{d}n}{n} = \frac{q^2\,\omega_0{}^2}{2v}\ln\frac{v^2}{m^{-1}\mathrm{k}T}\,. \qquad (22)$$

This is the polarization loss mentioned by Fermi [13] and Pines and Bohm [14].

For an electron the energy loss, Eq. (22), due to resonant interaction with the plasma as a continuous medium is less than one half of the energy loss Eq. (1).

$$\frac{e^2\,\omega_0{}^2}{v}\left[\ln\frac{l_D\mathrm{k}T}{e^2} + \ln\frac{v^2}{m^{-1}\mathrm{k}T}\right], \qquad (1\,\mathrm{a})$$

which is due to encounters with plasma electrons as particles.

The energy loss is not at all equal to the radiated energy from the charged particle, which will be found in the following to be zero. In fact the dielectric constant ε is not hermitian at $\omega^2 = \omega_0{}^2$, where collision effects can not be neglected.

By use of Eqs. (9), (11), (16), and (19) we can calculate the electric field at a position $\mathbf{r}$. On choosing the z-axis along the particle velocity $\mathbf{v}$ we let

$$\mathbf{v} = (0, 0, \beta c), \quad \mathbf{r} = (\varrho, 0, z), \qquad \boldsymbol{\varkappa} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)\,. \qquad (23)$$

Then we have

$$E_z(\mathbf{r},t) = -\frac{qv}{2\pi^2c^3}\,\mathrm{i}\iiiint\frac{n^2\cos^2\theta-\varepsilon}{\varepsilon\,(n^2-\varepsilon)}\,\delta\,(n\beta\cos\theta-1)$$

$$\exp\left[\mathrm{i}\frac{n\omega}{c}\,(\varrho\sin\theta\cos\varphi + z\cos\theta) - \mathrm{i}\omega t\right]\sin\theta\mathrm{d}\theta\mathrm{d}\varphi n^2\mathrm{d}n\omega\mathrm{d}\omega$$

$$= -\frac{qv}{\pi c^3}\,\mathrm{i}\iiint\frac{n^2\cos^2\theta-\varepsilon}{(n^2-\varepsilon)}\,\delta\,(n\beta\cos\theta-1)$$

$$J_0\left(\frac{n\omega}{c}\,\varrho\,\sin\theta\right)\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\sin\theta\,\mathrm{d}\theta\,n^2\,\mathrm{d}n\,\omega\,\mathrm{d}\omega$$

$$= -\frac{q\mathrm{i}}{\pi v^2}\iint\limits_{n>\beta^{-1}}\frac{1-\beta^2\varepsilon}{\varepsilon\,(n^2-\varepsilon)}\,J_0\left(\frac{\omega\varrho}{v}\sqrt{\beta^2n^2-1}\right)$$

$$\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\,n\,\mathrm{d}n\,\omega\,\mathrm{d}\omega\,. \qquad (24\,\mathrm{a})$$

On taking $\sqrt{\beta^2n^2-1}$ as an integration variable, we obtain

$$E_z(\mathbf{r},t) = -\frac{q}{\pi v^2}\,\mathrm{i}\int\frac{1-\beta^2\varepsilon}{\varepsilon}\,K_0(h\varrho)\,\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\omega\mathrm{d}\omega\,, \qquad (24\,\mathrm{b})$$

where $h = (1-\beta^2\varepsilon)^{\frac{1}{2}}\,|\omega|/v$ and K_n is the modified Bessel function of the second kind, for which the following formula holds:

$$\int\limits_0^\infty\frac{x^{n+1}}{x^2+y^2}\,J_n(a\,x)\,\mathrm{d}\,x = y^n\,K_n(ay)\,. \qquad (24\,\mathrm{c})$$

The φ-component of $\mathbf{E}$ is zero.

As regards the magnetic field, its φ-component is calculated to be

$$H_\varphi(\mathbf{r},t) = \frac{q}{\pi c}\int\limits_{-\infty}^{\infty}h\,K_1\,(h\,\varrho)\,\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\,\mathrm{d}\omega\,. \qquad (25)$$

The z- and ϱ-components are zero.

The power radiated from a cylindrical surface which is at the distance ϱ from the particle track is given by the surface integral of the Poynting vector

$$\int S_\varrho\,2\pi\,\varrho\,\mathrm{d}z = \frac{c}{4\pi}\int\limits_{\infty}^{\infty} -E_z H_\varphi 2\pi\varrho\,\mathrm{d}z$$

$$= \frac{q^2}{\pi v}\,\mathrm{i}\int\limits_{-\infty}^{\infty}\frac{1-\beta^2\varepsilon}{\varepsilon}\,h\varrho\,K_1\,(h\varrho)\,K_0\,(h\varrho)\,\omega\,\mathrm{d}\,\omega\,, \qquad (26)$$

which is calculated to be

$$\int S_\varrho\,2\pi\,\varrho\,\mathrm{d}z = \frac{q^2\,\omega_0{}^2}{v}\,\frac{\omega_0\varrho}{v}\,K_1\left(\frac{\omega_0\varrho}{v}\right)K_0\left(\frac{\omega_0\varrho}{v}\right). \qquad (27)$$

This result is in agreement with Eq. (22) when we let ϱ be l_{D}. However, it decreases exponentially for large ϱ,

$$\int S_\varrho\,2\pi\,\varrho\,\mathrm{d}z \sim \frac{\pi}{2}\,\frac{q^2\omega_0{}^2}{v}\exp\left(-2\,\omega_0\,\varrho/v\right), \qquad (28)$$

and hence radiation is not emitted beyond v/ω_0. The energy loss Eq. (22) is not in the form of radiation but in a kind of work $\oint\mathbf{E}\cdot\mathrm{d}\mathbf{D}$ done in the vicinity of the particle track.

4. Čerenkov radiation in collisionless plasmas

In this Section we consider the energy loss of a moving point charge due to the Čerenkov radiation.

Assumption 1. The collision frequency is negligible compared with the gyration frequency ω_{H} of the plasma electron; but the latter is not much larger than the plasma frequency ω_0,

$$\omega_{\mathrm{H}} \lesssim \omega_0\,. \qquad (29)$$

Assumption 2. The speed v of the moving point charge is much larger than thermal velocity of the plasma electron,

$$\ln\left(v^2/m^{-1}\,\mathrm{k}\,T\right) \gg 1\,, \qquad (30)$$

where k is the Boltzmann constant, T is the plasma temperature, and m is the electron mass.

Assumption 3. The radius of curvature of the orbit is much longer than the length passed in a period of plasma oscillation.

The last assumption enables us to consider the particle velocity as uniform in the time interval $1/\omega_0$, a characteristic time for the Čerenkov radiation, and to use Eq. (17).

The z-axis being chosen in the direction of $\mathbf{H}$, the dielectric tensor takes the form

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & \mathrm{i}\varepsilon_2 & 0 \\ -\mathrm{i}\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \tag{31}$$

where ε_1, ε_2 and ε_3 are real for a collisionless plasma. The determinant of the tensor T is calculated to be

$$\det \mathsf{T} = n^4 [\varepsilon_1 \sin^2\gamma + \varepsilon_3 \cos^2\gamma] - n^2 [\varepsilon_1 \varepsilon_3 (1+\cos^2\gamma) + (\varepsilon_1^2 - \varepsilon_2^2) \sin^2\gamma] + (\varepsilon_1^2 - \varepsilon_2^2)\, \varepsilon_3, \tag{32}$$

where γ is the angle between $\mathbf{H}$ and $\boldsymbol{\varkappa}$. Let the components of $\boldsymbol{\varkappa}$ be

$$\boldsymbol{\varkappa} = (\sin\gamma \cos\alpha, \sin\gamma \sin\alpha, \cos\gamma). \tag{33}$$

Then we have

$$\begin{aligned} \mathbf{v} \cdot \mathsf{T}^{-1} \cdot \mathbf{v} = (\det \mathsf{T})^{-1} v^2 \{ & n^4 \cos^2\theta \\ & - n^2 [\varepsilon_1 (1+\cos^2\theta) + (\varepsilon_3 - \varepsilon_1) \sin^2\Theta (1 - \sin^2\gamma \sin^2\alpha)] \\ & + \varepsilon_1 \varepsilon_3 \sin^2\Theta + (\varepsilon_1^2 - \varepsilon_2^2) \cos^2\Theta \}, \end{aligned} \tag{34a}$$

where Θ is the angle between $\mathbf{H}$ and $\mathbf{v}$, and θ is the angle between $\mathbf{v}$ and $\boldsymbol{\varkappa}$ so that

$$\cos\theta = \cos\Theta \cos\gamma - \sin\Theta \sin\gamma \cos\alpha. \tag{34b}$$

The integral with respect to βn can be cut off at $v/(|\omega|\, l_{\mathrm{D}})$, and the total integral is mostly contributed by the region of large βn, where $\det \mathsf{T}$ can be replaced by the first term in the right-hand side of Eq. (32) and $\cos^2\theta$ is very small because of the factor $\delta(n\beta\cos\theta - 1)$. Moreover, the nonvanishing integral with respect to n comes from the point where $\det \mathsf{T}$ is zero. From the inequalities

$$|\varepsilon_1 \sin^2\gamma + \varepsilon_3 \cos^2\gamma| \ll 1, |\cos\theta| \ll 1, \tag{34c}$$

it follows that

$$\begin{aligned} & |\varepsilon_1 (1+\cos^2\theta) + (\varepsilon_3 - \varepsilon_1) \sin^2\Theta (1 - \sin^2\gamma \sin^2\alpha)| \\ & |\varepsilon_1 \sin^2\gamma + \varepsilon_3 \cos^2\gamma + 2(\varepsilon_3 - \varepsilon_1) \cos\Theta \cos\gamma \cos\theta + \varepsilon_3 \cos^2\theta| \\ & \ll 1, \end{aligned} \tag{34d}$$

and the right-hand side of Eq. (34a) is approximated by its first term. Thus we obtain our basic relation

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{q^2 v^2}{2\pi^2 c^3}\, \mathrm{i} \int_{-\infty}^{\infty} \mathrm{d}\omega \int_0^{n_m} \mathrm{d}n \int \mathrm{d}\Omega \frac{\cos^2\theta}{\varepsilon_1 + (\varepsilon_3 - \varepsilon_1)\cos^2\gamma} \times \delta(n\beta\cos\theta - 1)\, n^2 \omega \tag{35}$$

$$n_m = c/(|\omega|\, l_{\mathrm{D}}).$$

As regards the integration with respect to the direction of $\boldsymbol{\varkappa}$, we introduce azimuthal angle φ of $\boldsymbol{\varkappa}$ around $\mathbf{v}$ so that

$$\mathrm{d}\Omega = \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\varphi, \quad \cos\gamma = \cos\Theta \cos\theta + \sin\Theta \sin\theta \cos\varphi, \tag{36}$$

Θ being the angle between $\mathbf{H}$ and $\mathbf{v}$.

The three components of the dielectric tensor are given by

$$\varepsilon_1 = 1 - \frac{\omega_0^2}{\omega^2 - \omega_{\mathrm{H}}^2}, \quad \varepsilon_2 = \frac{\omega_{\mathrm{H}}\, \omega_0^2}{\omega(\omega^2 - \omega_{\mathrm{H}}^2)}, \quad \varepsilon_3 = 1 - \frac{\omega_0^2}{\omega^2}, \tag{37}$$

for wavelength longer than the Debye length. The frequency of the dominant radiation is given by the inequalities (34c), namely

$$\left| \frac{\omega^2 (\omega^2 - \omega_0^2 - \omega_{\mathrm{H}}^2) + \omega_0^2 \omega_{\mathrm{H}}^2 \cos^2\gamma}{\omega^2 (\omega^2 - \omega_{\mathrm{H}}^2)} \right| \ll 1, \cos^2\theta \ll 1, \tag{38}$$

Hence the domain of ω^2 is composed of two parts

$$\begin{aligned} & 0 < \omega^2 < \mathrm{Min}\,(\omega_{\mathrm{H}}^2, \omega_0^2) \\ & \mathrm{Max}\,(\omega_{\mathrm{H}}^2, \omega_0^2) < \omega^2 < \omega_0^2 + \omega_{\mathrm{H}}^2. \end{aligned} \tag{39}$$

In particular,

$$\begin{aligned} & \omega^2 \approx \omega_0^2 + \omega_{\mathrm{H}}^2 \quad \text{for } \mathbf{v} \times \mathbf{H} = 0, \\ & \omega^2 \approx \omega_0^2 + \omega_{\mathrm{H}}^2 \quad \text{for } \mathbf{v} \cdot \mathbf{H} = \boldsymbol{\varkappa} \cdot \mathbf{H} = 0, \\ & \omega^2 \approx \omega_0^2 \quad \text{for } \mathbf{v} \cdot \mathbf{H} = 0, \quad \boldsymbol{\varkappa} \times \mathbf{H} \approx 0. \end{aligned} \tag{40}$$

(See Fig. 1). The set of intervals, Eq. (39), for the radiation frequency was pointed out by KOLOMENSKII [2] in the case of $\mathbf{v}$ parallel to $\mathbf{H}$.

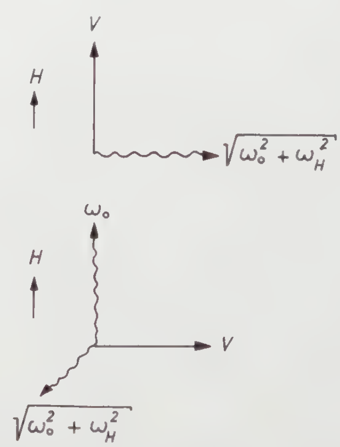

Fig. 1 Frequency of the dominant Cerenkov radiation, see Eq. (39).

The dielectric tensor $\boldsymbol{\varepsilon}(\omega)$ is hermitian excepting two points, $\omega^2 = 0$ and $\omega^2 = \omega_{\mathrm{H}}^2$, where the integrand of Eq. (35) vanishes. Therefore, according to one of the criteria mentioned in Section 2, the energy loss Eq. (35) is entirely due to radiated power.

On substituting Eq. (37) for ε_1 and ε_3 in (35) we consider the integral

$$I \equiv \frac{\mathrm{i}}{\pi} \int_{\omega > 0} \frac{\omega^2 (\omega^2 - \omega_{\mathrm{H}}^2)\, \mathrm{d}\omega^2}{\omega^2 (\omega^2 - \omega_{\mathrm{H}}^2 - \omega_0^2) + \omega_{\mathrm{H}}^2 \omega_0^2 \cos^2\gamma}. \tag{41}$$

Let us write the denominator in the form $(\omega^2-\omega'^2)(\omega^2-\omega''^2)$ for which

$$0<\omega'^2<\operatorname{Min}(\omega_H^2,\omega_0^2)\,,$$
$$\operatorname{Max}(\omega_H^2,\omega_0^2)<\omega''^2<\omega_0^2+\omega_H^2\,, \tag{42}$$
$$\omega'^2+\omega''^2=\omega_0^2+\omega_H^2\,.$$

Then we have

$$I=\frac{\omega'^2(\omega_H^2-\omega'^2)}{\omega''^2-\omega'^2}+\frac{\omega''^2(\omega''^2-\omega_H^2)}{\omega''^2-\omega'^2} \tag{43}$$

the first and second terms on the right corresponding to the first and second parts of the domain, Eq. (39), respectively. This expression is equal to $\omega'^2+\omega''^2-\omega_H^2$ or ω_0^2, and we finally obtain

$$-\frac{\mathrm{d}W}{\mathrm{d}t}=\frac{q^2\omega_0^2}{v}\int_0^{n_m}\int_0^{\pi}\beta^3\cos^2\theta\,\delta(n\beta\cos\theta-1)\,n^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}n$$
$$=\frac{q^2\omega_0^2}{v}\int_{1/\beta}^{c\,\omega_0 l_D}\frac{\mathrm{d}n}{n}=\frac{q^2\omega_0^2}{2v}\ln\frac{v^2}{m^{-1}kT}\,. \tag{44}$$

We have tacitly assumed that the dominant frequency of the Čerenkov radiation in a collisionless plasma is of the same order of magnitude as the plasma frequency ω_0. This is in fact true, because the first term of Eq. (43) is negligible when $\omega_H^2\ll\omega_0^2$. Thus we reach the following conclusions:

1. The power lost in the form of Čerenkov radiation is given by Eq. (44). It depends neither on the absolute value nor on the direction of the magnetic field **H**; it is just the same as the polarization loss, Eq. (22), in a plasma with no magnetic field.
2. The Čerenkov radiation is emitted in the direction almost perpendicular to the particle velocity. The power is uniform with respect to the azimuthal angle around the particle velocity.
3. The frequency ω is in the domain Eq. (39). The radiation in the first part is negligible if $\omega_H^2\ll\omega_0^2$.

5. Collisional damping of the Čerenkov radiation

Having discussed the Čerenkov radiation in a collisionless plasma, in this Section we consider damping of the radiation due to finite electric conductivity of the plasma.

Assumption 1. The collision frequency, ω_{coll}, is still much smaller than the gyration and plasma frequencies:

$$\omega_{\mathrm{coll}}^2\ll\operatorname{Min}(\omega_H^2,\omega_0^2)\,. \tag{45}$$

Assumption 2.

$$m^{-1}kT\ll v^2\ll c^2\,. \tag{46}$$

Assumption 3. The velocity **v** of the moving particle is parallel to the magnetic field **H**.

The first assumption will be taken into account when we make expansions in $\omega_{\mathrm{coll}}/\omega$. The first half of the second assumption corresponds to the relation that the wavelength is much larger than the Debye length; the second half enables us to treat large values of the refractive index only. For velocities which are not parallel to the magnetic field, calculation would be formidable.

We start from the expression, Eqs. (9) and (10), for the electric and magnetic fields with the electric current given by Eq. (16). On choosing the z-axis along the particle track, we let

$$\mathbf{v}=(0,0,\beta c),\qquad \mathbf{r}=(\varrho,0,z)\,,$$
$$\boldsymbol{\varkappa}=(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)\,. \tag{47}$$

Because of the δ-function large values of n need to be considered, for which

$$\mathbf{E}(\mathbf{r},t)=\frac{-2q}{(2\pi)^2c^2}\mathrm{i}\iiiint\frac{\varkappa\beta\cos\theta}{\varepsilon_1\sin^2\theta+\varepsilon_3\cos^2\theta}\,\delta(n\beta\cos\theta-1)$$
$$\exp\left(\mathrm{i}\frac{n\omega}{c}\varrho\sin\theta\cos\varphi\right)\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi\,n^2\,\mathrm{d}n\,\omega\,\mathrm{d}\omega\,. \tag{48}$$

Its z-component is calculated to be, Eq. (24b),

$$E_z(\mathbf{r},t)=-\frac{\mathrm{i}q}{\pi v^2}\int_{-\infty}^{\infty}\frac{1}{\varepsilon_1}K_0(h\varrho)\,\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\,\omega\,\mathrm{d}\omega\,, \tag{49a}$$

where

$$h^2=\frac{\varepsilon_3}{\varepsilon_1}\frac{\omega^2}{v^2}\,,\quad \operatorname{Re}h>0\,, \tag{49b}$$

and K_n is the modified Bessel function as in Section 3. The φ-component of **E** is negligible.

As regards the magnetic field **H**, its φ-component is found to be

$$H_\varphi(\mathbf{r},t)=\frac{q}{\pi c^2}\iiint\frac{\beta\varepsilon_1\sin\theta}{\varepsilon_1\sin^2\theta+\varepsilon_3\cos^2\theta}\,\delta(n\beta\cos\theta-1)$$
$$\times J_1\left(\frac{n\omega}{c}\varrho\sin\theta\right)\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\sin\theta\,\mathrm{d}\theta\,n\,\mathrm{d}n\,\omega\,\mathrm{d}\omega$$
$$=\frac{q}{\pi c}\int_{-\infty}^{\infty}h\,K_1(h\varrho)\,\mathrm{e}^{\mathrm{i}\omega(z/v-t)}\,\mathrm{d}\omega\,. \tag{50}$$

The power radiated from a cylindrical surface which is at the distance ϱ from the particle track is given by the surface integral of the Poynting vector, cf. Eq. (26),

$$\int_{-\infty}^{\infty}S_\varrho\,2\pi\varrho\,\mathrm{d}z=\frac{c}{4\pi}\int_{-\infty}^{\infty}-E_zH_\varphi\,2\pi\varrho\,\mathrm{d}z$$
$$=\frac{\mathrm{i}q^2}{\pi v}\int_{\infty}^{\infty}\frac{1}{\varepsilon_1}h^*\varrho\,K_1(h^*\varrho)\,K_0(h\varrho)\,\omega\,\mathrm{d}\omega\,. \tag{51}$$

For collisionless plasmas, the domain

$$0<\omega^2<\operatorname{Min}(\omega_H^2,\omega_0^2),$$
$$\operatorname{Max}(\omega_H^2,\omega_0^2)<\omega^2<\omega_0^2+\omega_H^2, \tag{52}$$

in which h is purely imaginary, contributes to the radiation. Regarding the collisional damping, we use the asymptotic formulas for large ϱ.

$$K_0(h\varrho) \sim (\pi/2h\varrho)^{\frac{1}{2}} e^{-h\varrho},$$
$$K_1(h^*\varrho) \sim (\pi/2h^*\varrho)^{\frac{1}{2}} e^{-h^*\varrho}. \quad (53a)$$

Then

$$h^*\varrho K_1(h^*\varrho) K_0(h\varrho) = \frac{\pi}{2}\frac{h^*}{|h|} e^{-2\varrho \mathrm{Re}\, h} \approx \pm i \frac{\pi}{2} e^{-2\varrho \mathrm{Re}\, h} \quad (53b)$$

because of the inequality $\mathrm{Re}\, h \ll |\mathrm{Im}\, h|$. Hence

$$\int S_\varrho 2\pi\varrho\, dz = \frac{q^2}{2v}\int \frac{\pm 1}{\varepsilon_1} e^{-2\varrho \mathrm{Re}\, h}\omega\, d\omega, \quad (54)$$

where $\pm$ is to be taken so that $\mathrm{Re}(\pm 1/\varepsilon_1) > 0$. Considering the relation (4), $\varepsilon_1(\omega) = \varepsilon_1^*(-\omega)$, we finally obtain

$$\int S_\varrho 2\pi\varrho\, dz = \frac{q^2}{v}\int_{\omega>0} \left| \mathrm{Re}\frac{1}{\varepsilon_1}\right| e^{-2\varrho \mathrm{Re}\, h}\omega\, d\omega. \quad (55)$$

By use of an effective collision frequency ω_{coll} let us adopt the expressions

$$\varepsilon_1 = 1 - \frac{\omega_0^2(\omega + i\omega_{\text{coll}})}{[(\omega + i\omega_{\text{coll}})^2 - \omega_H^2]},$$
$$\varepsilon_3 = 1 - \frac{\omega_0^2}{\omega(\omega + i\omega_{\text{coll}})}. \quad (56)$$

For a small ω_{coll} the real part of h is calculated, in the domain (52), to be

$$\mathrm{Re}\, h = \frac{\omega_{\text{coll}}}{2v}\left[\frac{(\omega^2-\omega_0^2)(\omega^2-\omega_H^2)}{\omega^2(\omega_0^2+\omega_H^2-\omega^2)}\right]^{\frac{1}{2}} \left|\frac{\omega_0^2}{\omega^2-\omega_0^2} + \frac{\omega_0^2(\omega^2+\omega_H)}{(\omega^2-\omega_H^2)(\omega_0^2+\omega_H^2-\omega^2)}\right|. \quad (57)$$

The method of steepest descent (Sattelpunktmethode) enables us to calculate Eq. (55). It is found that $\mathrm{Re}\, h$ takes its absolute minimum near the midpoint in the first part of the domain (52):

$$0 < \omega^2 < \mathrm{Min}(\omega_H^2, \omega_0^2). \quad (58)$$

In fact, the values of ω^2 which minimize $\mathrm{Re}\, h$ are the following:

$$\begin{aligned}
\omega^2 &= \tfrac{1}{2}\omega_H^2 && \text{for } \omega_H^2 \ll \omega_0^2\\
\omega^2 &= 0.479\,\omega_H^2 && \text{for } \omega_H^2 = \tfrac{1}{2}\omega_0^2\\
\omega^2 &= \tfrac{1}{2}\omega_H^2 = \tfrac{1}{2}\omega_0^2 && \text{for } \omega_H^2 = \omega_0^2 \qquad (59)\\
\omega^2 &= 0.574\,\omega_0^2 && \text{for } \omega_H^2 = 2\omega_0^2\\
\omega^2 &= 0.560\,\omega_0^2 && \text{for } \omega_H^2 = 3\omega_0^2\\
\omega^2 &= \tfrac{1}{2}\omega_0^2 && \text{for } \omega_H^2 \gg \omega_0^2.
\end{aligned}$$

The expression of Eq. (55) is calculated to be

$$\int S_\varrho 2\pi\varrho\, dz = \frac{1}{8}\left(\frac{\pi}{2\omega_{\text{coll}} v\varrho}\right)^{\frac{1}{2}} q^2 \frac{\omega_H^4}{\omega_0^2}\exp(-4\omega_{\text{coll}}\varrho/v) \quad \text{for } \omega_H^2 \ll \omega_0^2 \quad (60a)$$

$$= \frac{1}{8}\left(\frac{\pi}{\sqrt{3}\,\omega_{\text{coll}} v\varrho}\right)^{\frac{1}{2}} q^2\omega_0^2 \exp(-4\omega_{\text{coll}}\varrho/\sqrt{3}\,v) \quad \text{for } \omega_H^2 = \omega_0^2 \quad (60b)$$

$$= \frac{1}{4}\left(\frac{\pi}{\omega_{\text{coll}} v\varrho}\right)^{\frac{1}{2}} q^2\omega_0^2 \exp(-2\omega_{\text{coll}}\varrho/v) \quad \text{for } \omega_H^2 \gg \omega_0^2. \quad (60c)$$

6. Non-relativistic cyclotron radiation in collisionless plasmas

A characteristic feature of the non-relativistic cyclotron radiation from a point charge q in a collisionless plasma is that the variation of $\mathbf{k}\cdot\mathbf{r}_e$ in Eq. (12) is negligible (for k not too large) in comparison with the variation of ωt:

$$\mathbf{J} = \frac{q}{(2\pi)^4}\int \mathbf{v}\, e^{i\omega t}\, dt. \quad (61)$$

On choosing the z-axis in the direction of the magnetic field, we let

$$\omega_1 = |q|\, H/m_1 c$$
$$\mathbf{v} = (\pm v_1 \sin\omega_1 t, v_1\cos\omega_1 t, v_2), \quad (62)$$

where m_1 is the particle mass and the upper and the lower signs of $\pm$ are for positive and negative q. From Eq. (61) we then obtain

$$J_x = \frac{\pm q v_1}{(2\pi)^4}\int \frac{e^{i\omega_1 t} - e^{-i\omega_1 t}}{2i} e^{i\omega t}\, dt = \pm i\frac{q v_1}{(2\pi)^3}\frac{1}{2}[\delta(\omega-\omega_1) - \delta(\omega+\omega_1)],$$
$$J_y = \frac{q v_1}{(2\pi)^3}\frac{1}{2}[\delta(\omega-\omega_1) + \delta(\omega+\omega_1)],$$
$$J_z = \frac{q v_2}{(2\pi)^3}\delta(\omega). \quad (63)$$

The Eq. (13) with Eq. (11) is similarly reduced to

$$-\frac{dW}{dt} = \frac{4\pi q}{c^3} i \int\int\int \mathbf{v}\cdot \mathsf{T}^{-1}(\omega)\cdot \mathbf{J} e^{-i\omega t} d\Omega\, n^2 dn\, |\omega|\, \omega\, d\omega. \quad (64)$$

By use of the vector

$$\mathbf{a} \equiv (a_x, a_y, a_z) = \left(\pm\frac{1}{2}i, \frac{1}{2}, 0\right), \quad (65)$$

$\omega\mathbf{J}$ in Eq. (64) can be written as

$$\omega\mathbf{J} = (2\pi)^{-3} q v_1 \omega_1[\mathbf{a}\,\delta(\omega-\omega_1) - \mathbf{a}^*\delta(\omega+\omega_1)] \quad (66)$$

since $\omega\delta(\omega) = 0$; and we have

$$-\frac{dW}{dt} = \frac{q^2 v_1 \omega_1^2}{2\pi^2 c^3} i \int\int [\mathbf{v}\cdot\mathsf{T}^{-1}(\omega_1)\cdot\mathbf{a}\, e^{-i\omega_1 t} - \mathbf{v}\cdot\mathsf{T}^{-1}(-\omega_1)\cdot\mathbf{a}^* e^{i\omega_1 t}]\, n^2\, dn\, d\Omega. \quad (67)$$

On taking the time average over a gyration period,

$$\langle \mathbf{v}\, e^{-i\omega_1 t}\rangle_{AV} = v_1\mathbf{a}^*, \quad \langle \mathbf{v}\, e^{i\omega_1 t}\rangle_{AV} = v_1\mathbf{a}, \quad (68)$$

and making use of the relation, see Eq. (6),

$$\mathbf{a}\cdot\mathsf{T}^{-1}(-\omega_1)\cdot\mathbf{a}^* = [\mathbf{a}^*\cdot\mathsf{T}^{-1}(\omega_1)\cdot\mathbf{a}]^*, \quad (69)$$

we finally obtain

$$-\frac{dW}{dt} = \frac{2}{\pi}\frac{q^2 v_1^2 \omega_1^2}{c^3}\,\mathrm{Re}\, i \int_0^{\pi}\int_0^{\infty} \mathbf{a}^* \cdot \mathsf{T}^{-1}(\omega_1)\cdot \mathbf{a}\, n^2\, dn \sin\gamma\, d\gamma, \tag{70}$$

where γ is the angle between the magnetic field and wave vector.

When the wave vector is taken in the yz-plane, the tensor T is of the form

$$\mathsf{T} = \begin{pmatrix} -n^2+\varepsilon_1 & i\,\varepsilon_2 & 0 \\ -i\,\varepsilon_2 & -n^2\sin^2\gamma+\varepsilon_1 & n^2\sin\gamma\cos\gamma \\ 0 & n^2\sin\gamma\cos\gamma & -n^2\sin^2\gamma+\varepsilon_3 \end{pmatrix}, \quad \varepsilon_2 > 0, \tag{71}$$

for which we get

$$\begin{aligned} \mathbf{a}^*\cdot\mathsf{T}^{-1}\cdot\mathbf{a}\,\det\mathsf{T} &= |a_x|^2\,[-n^2(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)+\varepsilon_1\varepsilon_3] \\ &\quad \pm 2\,\varepsilon_2\,|a_x|\,a_y\,(n^2\sin^2\gamma-\varepsilon_3) \\ &\quad - a_y^2(-n^2+\varepsilon_1)(-n^2\sin^2\gamma+\varepsilon_3)\,. \end{aligned} \tag{72}$$

The discriminant of the right-hand side of this equation as a quadratic form of $|a_x|$ and a_y is just the $\det\mathsf{T}$ given by Eq. (32). Let

$$\det\mathsf{T} \equiv (\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)\,(n^2-n_1^2)\,(n^2-n_2^2)\,, \tag{73}$$

where n_1^2 and n_2^2 are real, then for $n^2 = n_j^2$ $(j=1,2)$ the right hand side of Eq. (70) is

$$\begin{aligned} &[-n_j^2(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)+\varepsilon_1\varepsilon_3]\,(|a_x|\pm\alpha_j a_y)^2 \\ &=[-n_j^2(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)+\varepsilon_1\varepsilon_3]\,[(1\pm\alpha_j)/2]^2, \end{aligned} \tag{74a}$$

where

$$\alpha_j = -\frac{\varepsilon_2(-n_j^2\sin^2\gamma+\varepsilon_3)}{-n_j^2(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)+\varepsilon_1\varepsilon_3} \tag{74b}$$

On carrying out the integration with respect to n in Eq. (70), we have

$$\begin{aligned} -\frac{dW}{dt} = \frac{q^2 v_1^2\omega_1^2}{c^3}\int_0^{\pi}\sum_{j=1,2}\Bigg\{\left(\frac{1\pm\alpha_j}{2}\right)^2 \\ \times \left|\frac{-n_j^2(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)+\varepsilon_1\varepsilon_3}{(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)(n_1^2-n_2^2)}\mathrm{Re}\, n_j\right|\Bigg\}_{\omega=\omega_1}\sin\gamma\, d\gamma\,. \end{aligned} \tag{75}$$

This integral comes entirely from real positive poles of the integrand of Eq. (64) or from real positive roots of $\det\mathsf{T}=0$. Hence, according to the second criterion in Section 2, the energy loss, Eq. (75), is due to the radiation with these indexes of refraction.

The ε_1, ε_2 and ε_3 are given by Eq. (37); n_1^2 and n_2^2 are given by Appleton-Hartree's formula (see, e.g., [15]):

$$\begin{aligned} n_j^2 = 1 - 2\omega_0^2(\omega^2-\omega_0^2)/\{2\,\omega^2(\omega^2-\omega_0^2) - \omega^2\omega_H^2\sin^2\gamma \\ +(-1)^{j-1}[\omega^4\omega_H^4\sin^4\gamma + 4\,\omega^2\omega_H^2(\omega^2-\omega_0^2)^2\cos^2\gamma]^{\frac{1}{2}}\}, \end{aligned} \tag{76}$$

where $(-1)^{j-1}=1$ for the ordinary wave and $(-1)^{j-1}=-1$ for the extraordinary wave; and

$$\left|\frac{-n_j^2(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)+\varepsilon_1\varepsilon_3}{(\varepsilon_1\sin^2\gamma+\varepsilon_3\cos^2\gamma)(n_1^2-n_2^2)}\right| = \frac{1}{1+K_j^2}\,,$$

$$K_j = \frac{\omega\,\omega_H\sin^2\gamma+(-1)^{j-1}[\omega^2\omega_H^2\sin^4\gamma+4(\omega^2-\omega_0^2)^2\cos^2\gamma]^{\frac{1}{2}}}{2(\omega^2-\omega_0^2)\cos\gamma}\,. \tag{77}$$

Let us first consider an electron $(q=-e)$ which is gyrating in a dilute plasma under the conditions

$$m^{-1}kT \ll v^2 \ll c^2, \quad \omega_0^2 \ll \omega_H^2\,. \tag{78}$$

The gyration frequency ω_1 is then given by

$$\omega_1^2 = \omega_H^2\,(1-v^2/c^2)\,, \tag{79}$$

and furthermore, for $\omega=\omega_1$,

$$\begin{aligned} &K_1 = 1/\cos\gamma\,, && K_2 = -\cos\gamma\,, \\ &n_1 = 1\,, && n_2^2 = \frac{2s+1}{s\sin^2\gamma+1}\,, \\ &\alpha_1 = 1\,, && \alpha_2 = \frac{s\sin^2\gamma-\cos^2\gamma}{s\sin^2\gamma+1}\,, \\ &s \equiv (c^2/v^2)\,(\omega_0^2/\omega_H^2)\,. \end{aligned} \tag{80}$$

By use of these relationships, the radiated power is found to be

$$-\frac{dW}{dt} = \frac{e^2\omega_1^2 v_1^2}{c^3} f_e(s)\,, \tag{81a}$$

$$f_e(s) = \int_0^{\pi}\left(\frac{2s+1}{s\sin^2\gamma+1}\right)^{\frac{1}{2}}\frac{1+\cos^2\gamma}{4(s\sin^2\gamma+1)^2}\sin\gamma\,d\gamma = \frac{(s+2)\sqrt{2s+1}}{3(s+1)^2}\,. \tag{81b}$$

The function $f_e(s)$ decreases from $f_e(0)=2/3$, the well-known value for vacuum, to $f_e(\infty)=0$ as the plasma density increases. (Fig. 2). For ω_0^2 not much smaller than ω_H^2, no radiation is emitted from an electron since $\alpha_1=\alpha_2=1$, as previously pointed out by GINZBURG [6].

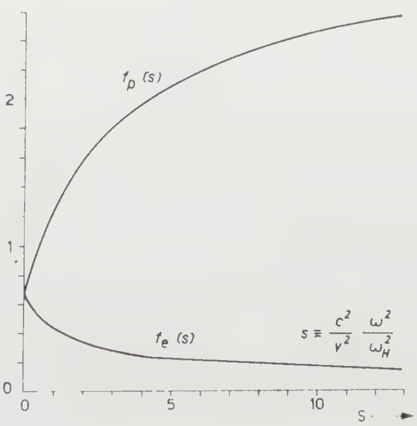

Fig. 2 Cyclotron radiation from an electron, f_e, and from a positron, f_p, see Eqs. (81), (82).

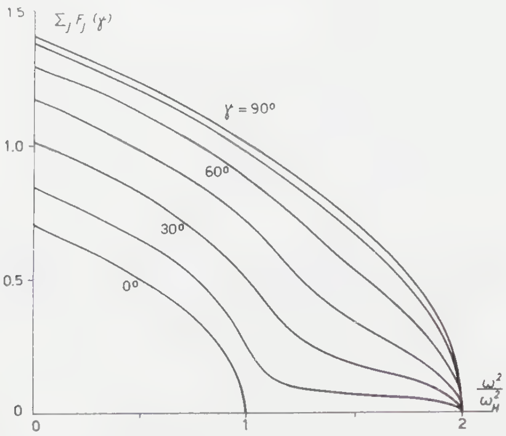

Fig. 3 Angular distribution of the cyclotron radiation from a positron in the non-relativistic limit, see Eq. (83).

It is interesting to compare a positron ($q = +e$) with an electron ($q = -e$). The radiated power from a gyrating positron under the condition (78) is calculated to be

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{e^2\,\omega_1{}^2\,v_1{}^2}{c^3} f_{\mathrm{P}}(s)\,, \tag{82a}$$

$$f_{\mathrm{P}}(s) = 2 - \frac{\pi}{2} + 2\tan^{-1}\sqrt{2s+1} - \frac{(5s+4)\sqrt{2s+1}}{3\,(s+1)^2}\,. \tag{82b}$$

The function $f_{\mathrm{P}}(s)$ increases from $f_{\mathrm{P}}(0) = 2/3$ to $f_{\mathrm{P}}(\infty) = 2 + \pi/2$ as the plasma density increases (Fig. 2). For $\omega_0{}^2$ not much smaller than $\omega_{\mathrm{H}}{}^2$, the cyclotron radiation power is given by

$$-\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{e^2\,\omega_{\mathrm{H}}{}^2\,v_1{}^2}{c^3} \int_0^{\pi} \sum_{j=1}^{2} F_j(\gamma)\,\mathrm{d}\gamma\,, \tag{83a}$$

where

$$F_j(\gamma) = \left(\frac{|\operatorname{Re} n_j|}{1 + K_j{}^2}\right)_{\omega = \omega_{\mathrm{H}}} \sin\gamma\,. \tag{83b}$$

The integrand $\sum F_j(\gamma)$ is shown in Fig. 3 as a function of $\omega_0{}^2/\omega_{\mathrm{H}}{}^2$ and γ. Fig. 4 shows the integrals $\int F_j(\gamma)\,\mathrm{d}\gamma$ which take, in particular, the following values:

$$\int F_1(\gamma)\,\mathrm{d}\gamma = 2 - \pi/2, \qquad \int F_2(\gamma)\,\mathrm{d}\gamma = \pi \qquad \text{for } v^2/c^2 \ll \omega_0{}^2/\omega^2 \ll 1\,;$$

$$\int F_1(\gamma)\,\mathrm{d}\gamma = 0\,, \qquad \int F_2(\gamma)\,\mathrm{d}\gamma = 2 \qquad \text{for } \omega_0{}^2/\omega_{\mathrm{H}}{}^2 = 1\,;$$

$$\int F_1(\gamma)\,\mathrm{d}\gamma = 0\,, \qquad \int F_2(\gamma)\,\mathrm{d}\gamma = 0 \qquad \text{for } \omega_0{}^2/\omega_{\mathrm{H}}{}^2 \geq 2. \tag{84}$$

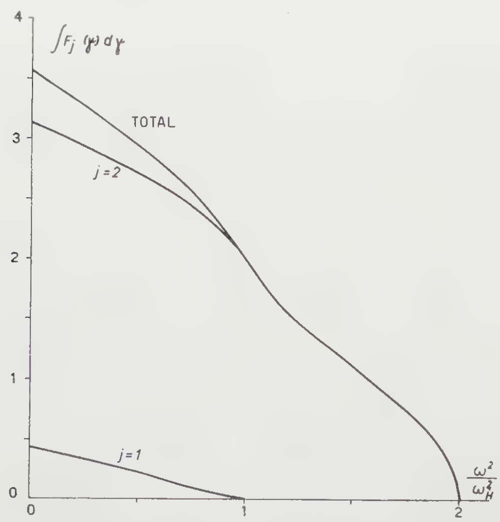

Fig. 4. Integrated cyclotron radiation from a positron in the non-relativistic limit, see Eq. (83).

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ЦИКЛОТРОННОЕ ИЗЛУЧЕНИЕ ИОНОВ В ПЛАЗМЕ

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ОРДЕНА ЛЕНИНА ИНСТИТУТ АТОМНОЙ ЭНЕРГИИ ИМ. И. В. КУРЧАТОВА

АКАДЕМИИ НАУК СССР, МОСКВА, СССР

В работе определена интенсивность излучения быстрых ионов в холодной плазме. Эта задача возникла в связи с наблюдением на установке Огра в определенных режимах резонансных пиков напряженности электрического поля на циклотронной ионной частоте и ее обертонах. Хотя указанные наблюдения производились не в волновой зоне, естественно ожидать, что обнаруженная в эксперименте зависимость числа наблюдаемых пиков от плотности холодной плазмы должна найти свое отражение и в интенсивности излучения. Расчеты показали, что при увеличении скорости ионов и плотности плазмы максимум интенсивности излучения, действительно, смещается в сторону высоких частот.

Это смещение аналогично смещению максимума интенсивности излучения при синхротронном излучении электрона, имеющего скорость, близкую к скорости света. В случае ионов в плазме роль скорости света играет фазовая скорость электромагнитных волн в плазме, которая в существенной для расчетов области близка к альфвеновской скорости $c_A = B_0/(4\pi\, m_i\, n_0)^{1/2}$. Поэтому высокие обертоны проявляются уже при сравнительно небольшой скорости ионов $v \sim c_A \ll c$.

1. Введение

В настоящей работе получены формулы для определения интенсивности излучения быстрых ионов, движущихся в холодной плазме, находящейся в магнитом поле. Задача об излучении ионов возникла в связи с наблюдением на установке Огра резонансных пиков напряженности электрического поля на циклотронной частоте и ее обертонах [1]. (Режимы, в которых наблюдались интенсивные пики напряженности электрического поля на обертонах ионной циклотронной частоты, получались при напуске в камеру Огры аргона до давления $p = 10^{-6}$-10^{-5} мм. рт.ст.). Хотя указанные наблюдения производились не в волновой зоне, так что для объяснения эффекта необходимо рассчитать средний квадрат напряженности квазистационарной части флюктуационного электрического поля, тем не менее естественно в качестве первого шага решить более простую задачу определения интенсивности излучения. Эта задача, вероятно, может представлять и самостоятельный интерес для диагностики плазмы в лабораторных условиях и при исследовании радиоизлучения из космоса.

Излучение ионов в плазме, находящейся в магнитном поле, отличается рядом особенностей, связанных с большой величиной показателя преломления такой плазмы. Как известно, в области низких частот порядка ионной циклотронной частоты показатель преломления плазмы равен примерно отношению скорости света к альфвеновской скорости $N \sim c/c_A$. В большинстве известных случаев для плазмы, находящейся в магнитном поле, $c_A \ll c$, так что $N \gg 1$. Но уже интенсивность дипольного излучения заряда в среде с показателем преломления N в N раз больше интенсивности излучения заряда в вакууме ([2], стр. 367). Если же скорость иона близка к фазовой скорости волны $v \sim c/N$, то интенсивность излучения может оказаться гораздо больше интенсивности дипольного излучения.* Как видно из приведенных ниже расчетов ионы могут излучать не меньше, чем электроны, движущиеся с той же скоростью в вакууме. Большая величина показателя преломления N существенно сказывается и на спектральном распределении интенсивности излучения. Из теории синхротронного излучения [3] известно, что при приближении скорости заряда к скорости света максимум интенсивности излучения смещается с циклотронной частоты $\omega_{Be} = |e|\, B/(m_e\, c)$, на ее обертоны $\omega = m\, \omega_{Be}$, причем практически достигаются значения $m \sim 10^5$. Подчеркнем, что этот эффект зависит от отношения скорости заряда к фазовой скорости электромагнитных волн и не связан с релятивизмом. Поэтому при $N \gg 1$, когда фазовая скорость волн $V = c/N \ll c$, смещение максимума интенсивности следует ожидать даже при скорости заряда, весьма далекой от релятивистской $v \ll c$. Конечно, эффект смещения максимума интенсивности магнитного излучения ионов не может быть таким же сильным, как при синхротронном излучении электрона, так как область частот, где $N \gg 1$, в плазме ограничена. Представляют интерес поэтому частоты $\omega = m\, \omega_{Bi}$ ($m = 1, 2, 3 \ldots$) со сравнительно небольшим номером m.

Излучение заряда в магнитном поле в среде с показателем преломления $N > 1$ исследовалось Цытовичем [4], который, однако, предполагал среду изотропной. Излучению заряда в плазме

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* В изотропной среде с показателем преломления N глобальная интенсивность излучения заряда, движущегося по окружности, равна

$$W = \frac{2}{3}\, \frac{e^2}{c^3}\, \frac{\omega_B^2\, v^2\, N}{[1 - (v\, N/c)^2]^2}$$

посвящены работы Эйдмана [5] и Твисса и Робертса [6]. Однако, их формулы приспособлены только к определению излучения электронов. В следующем разделе приводится кратко метод расчета излучения заряда без конкретизации вида тензора электрической проницаемости. Затем формулы применяются к случаю, когда излучателями являются ионы ($m_i \gg m_e$).

В настоящей работе предполагается, что эти ионы движутся в холодной плазме. Это означает, что их скорость v значительно превышает тепловую скорость ионов плазмы $v \gg v_T$. При этих условиях специфическое резонансное поглощение излучения в плазме будет незначительно. Столкновительным поглощением излучения мы также будем пренебрегать, считая, что плотность плазмы достаточно мала.

2. Метод расчета

Рассмотрим однородную среду (плазму), в которой движутся заряды с однородной плотностью n_1, (быстрые ионы). Потери энергии движущимися зарядами, создающими в среде ток с плотностью $\mathbf{j}$ и поле $\mathbf{E}$, равны работе поля над зарядами $\mathbf{j}\cdot\mathbf{E}$ (работа силы лучистого трения), взятой с обратным знаком. Метод расчета электрического поля и потерь энергии в анизотропной среде изложен в работах Гинзбурга [7] и др. [8], [9]. Для удобства изложения приведем его кратко в слегка измененной форме.

Разложим все величины $\mathbf{E}$, $\mathbf{j}$ и др. по плоским волнам

$$\mathbf{E}(\mathbf{r}, t) = \int \mathbf{E}_{\mathbf{k},\omega} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} d\mathbf{k} d\omega . \quad (1)$$

Для амплитуд $\mathbf{E}_{\mathbf{k},\omega}$, $\mathbf{j}_{\mathbf{k},\omega}$ из уравнений Максвелла получатся соотношения

$$\left\{\frac{k^2c^2}{\omega^2}(\delta_{\alpha\beta} - n_\alpha n_\beta) - \varepsilon_{\alpha\beta}\right\} E_\beta = i\,\frac{4\pi}{\omega} j_\alpha , \left(\mathbf{n} = \frac{\mathbf{k}}{k}\right). \quad (2)$$

Решение уравнений (2) ищем в виде

$$\mathbf{E} = \sum_{l=1}^{2} E_l \mathbf{a}_l , \quad (3)$$

где $\mathbf{a}_l$ — собственные векторы поля, удовлетворяющие однородной системе уравнений

$$\{N_l^2(\delta_{\alpha\beta} - n_\alpha n_\beta) - \varepsilon_{\alpha\beta}\} a_{l\alpha} = 0 . \quad (4)$$

Т.к. мы пренебрегаем поглощением, то тензор $\varepsilon_{\alpha\beta}$ эрмитов ($\varepsilon_{\alpha\beta} = \varepsilon_{\beta\alpha}{}^*$) и из (4) следует также уравнение

$$\{N_m^2(\delta_{\alpha\beta} - n_\alpha n_\beta) - \varepsilon_{\beta\alpha}\} a^*_{m\alpha} = 0 . \quad (4a)$$

Условие обращения в нуль детермината системы (4) (или 4a), определяет, как известно, два значения квадрата показателя преломления N_l^2 ($l = 1, 2$) которым соответствуют два собственных вектора $\mathbf{a}_l$. Умножая (4) на $a^*_{m\beta}$, а (4a) на $a_{l\beta}$ и вычитая затем (4a) из (4), получим

$$(\delta_{\alpha\beta} - n_\alpha n_\beta) a^*_{m\beta} a_{l\alpha} (N_l^2 - N_m^2) = 0 . \quad (5)$$

При $l \neq m$, множитель перед $N_l^2 - N_m^2$ обращается в нуль, при $l = m$ положим его равным единице. Таким образом, имеем следующее условие ортонормировки векторов $\mathbf{a}_l$:

$$a^*_{m\beta} a_{l\alpha} (\delta_{\alpha\beta} - n_\alpha n_\beta) = \delta_{lm} , \quad (6)$$

$$\varepsilon_{\alpha\beta} a_{l\alpha} a^*_{m\beta} = N_l^2 \delta_{lm} . \quad (6a)$$

Второе из условий есть следствие (6) и (4).

Подставляя теперь (3) в (2) и используя условие ортонормировки, находим $\mathbf{E}_{\mathbf{k},\omega}$ в виде

$$E_\alpha = L_{\alpha\beta} j_\beta , \quad L_{\alpha\beta} = \sum_{l=1}^{2} \frac{4\pi i}{\omega} \frac{a_{l\alpha} a^*_{l\beta}}{(kc/\omega)^2 - N_l^2} . \quad (7)$$

Выражение для средних потерь энергии, отнесенных к единице объема $Q = -\mathbf{j}(\mathbf{r}, t)\cdot\mathbf{E}(\mathbf{r}, t)$ получается в виде интеграла, в который входит корреляционная функция фурье-компонент плотности тока излучателей

$$j_{\alpha\mathbf{k},\omega} j^*_{\beta\mathbf{k},\omega} = G_{\alpha\beta}(\mathbf{k}, \omega)\,\delta(\mathbf{k} - \mathbf{k}')\,\delta(\omega - \omega') . \quad (8)$$

В приближении некоррелирующих зарядов, которым мы только и будем пользоваться, функция $G_{\alpha\beta}$ пропорциональна плотности зарядов и равна [10]

$$G_{\alpha\beta} = n_1 G^1_{\alpha\beta} ,$$

$$G^1_{\alpha\beta} = \frac{e^2}{(2\pi)^4} \left\langle \int_{-\infty}^{\infty} v_\alpha(t)\, v_\beta(0)\, e^{i(\omega t - \mathbf{k}\int_0^t \mathbf{v}(t')\,dt')} dt \right\rangle \quad (9)$$

Здесь $\mathbf{v}(t)$ — закон движения заряда, угловые скобки означают усреднение по начальным скоростям $\mathbf{v}(0)$.

Если все заряды имеют одинаковую скорость, то угловые скобки нужно опустить. Рассмотрим этот случай.

Подставляя (8) в выражение для $Q = -\mathbf{j}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)$, находим для потерь энергии одним зарядом:

$$-\frac{dE}{dt} = \frac{Q}{n_1} = -\int G^1_{\beta\alpha} L_{\alpha\beta}\, d\mathbf{k}\, d\omega \quad (10)$$

Здесь пределы интегрирования бесконечные. Учитывая, что по определению (8) $G_{\alpha\beta} = G^*_{\beta\alpha}$, а также, что согласно (9) $G_{\alpha\beta}(-\mathbf{k}, -\omega) = G^*_{\alpha\beta}(\mathbf{k}, \omega)$ и аналогично $L_{\alpha\beta}(-\mathbf{k}, -\omega) = L^*_{\alpha\beta}(\mathbf{k}, \omega)$ (вследствие $\varepsilon_{\alpha\beta}(-\mathbf{k}, -\omega) = \varepsilon^*_{\alpha\beta}(\mathbf{k}, \omega)$), можно записать (10) в виде интеграла по положительным частотам

$$-\frac{dE}{dt} = -2\,\mathrm{Re}\int_0^\infty d\omega \int G^1_{\beta\alpha} L_{\alpha\beta}\, d\mathbf{k} . \quad (11)$$

Вещественная часть интеграла получается при взятии вычетов в полюсах подинтегрального выражения, точнее в полюсах $L_{\alpha\beta}$ (т.к. $G_{\beta\alpha}$ не имеет полюсов). Функция $L_{\alpha\beta}$ имеет полюсы а) при $(kc/\omega)^2 = N_l^2$, вычеты в этих точках соответствуют излучению обыкновенной и необыкновенной волн; б) при $k \to \infty$ и $\varepsilon_{zz} = 0$ (ось z совпадает с направлением $\mathbf{k}$), вычеты в этих полюсах соответствуют излучению продольных волн, или, что то же самое,

поляризационным потерям заряда, которые нас здесь не интересуют.

Заметим, что вместо взятия вычетов, можно выделить δ-образную эрмитову часть тензора $L_{\alpha\beta}$ и тогда записать (11) в виде

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = -2\int\limits_0 \mathrm{d}\omega \int G^1{}_{\beta\alpha}\, L'_{\alpha\beta}\,\mathrm{d}\mathbf{k}, \quad L'_{\alpha\beta} = \frac{1}{2}\,(L_{\alpha\beta} + L^*_{\beta\alpha}) \tag{11a}$$

Ограничиваясь рассмотрением потерь на излучение, предположим, что среда обладает бесконечно малым поглощением ($\mathrm{Im}\, N_l^2 = \alpha > 0$). Тогда знаменатель в $L_{\alpha\beta}$ имеет вид

$$\frac{1}{x - \mathrm{i}\alpha} = \frac{x}{x^2+\alpha^2} + \mathrm{i}\,\frac{\alpha}{x^2+\alpha^2} \xrightarrow[(\alpha\to 0)]{} \frac{\mathsf{P}}{x} + \mathrm{i}\pi\delta(x), \tag{11b}$$

где P — оператор „главное значение“, $\delta(x)$ — δ-функция.

Таким образом,

$$L'_{\alpha\beta} = -\sum_{l=1}^{2} \frac{4\pi^2}{\omega}\, a_{l\alpha} a^*_{l\beta}\, \delta\left(\frac{k^2c^2}{\omega^2} - N_l^2\right). \tag{12}$$

Из выражений (11a) и (12) следует, что поскольку $(kc/\omega)^2 > 0$, излучение происходит в тех областях частот и углов, где $N_l^2 > 0$.

Входящий в выражение (11a) тензор $G_{\beta\alpha}$ удобно вычислить в системе координат, где постоянное магнитное поле $\mathbf{B}_0$ направлено по оси z_0, а вектор $\mathbf{k}$ лежит в плоскости $x_0\, z_0$. Опуская простые выкладки, приведем результат

$$G^1_{\beta\alpha} = \frac{e^2}{8\pi^3} \sum_{m=-\infty} \Pi^{(m)}_{\beta\alpha}\, \delta(\omega - k_\parallel v_\parallel - m\,\omega_B) \tag{13}$$

$$\Pi^{(m)}_{x_0x_0} = v_\perp{}^2\, \frac{m^2}{\lambda^2}\, J_m{}^2(\lambda)\,,$$

$$\Pi^{(m)}_{x_0y_0} = -\Pi^{(m)}_{y_0x_0} = \mathrm{i}\, v_\perp{}^2\, \frac{m}{\lambda}\, J_m(\lambda) J_m{}'(\lambda)\,,$$

$$\Pi^{(m)}_{y_0y_0} = v_\perp{}^2\, J_m{}'^2(\lambda)\,,$$

$$\Pi^{(m)}_{y_0z_0} = -\Pi^{(m)}_{z_0y_0} = -\mathrm{i}\, v_\perp v_\parallel\, \frac{m}{\lambda}\, J_m(\lambda)\, J_m{}'(\lambda)\,,$$

$$\Pi^{(m)}_{z_0z_0} = v_\parallel{}^2\, J_m{}^2(\lambda)\,,$$

$$\Pi^{(m)}_{x_0z_0} = \Pi^{(m)}_{z_0x_0} = v_\perp v_\parallel\, \frac{m}{\lambda}\, J_m{}^2(\lambda)\,. \tag{14}$$

Здесь $v_\perp$ и $v_\parallel$ поперечная и продольная (относительно постоянного магнитного поля) компоненты скорости заряда-излучателя; $k_\perp$, k — соответствующие компоненты волнового вентора $\mathbf{k}$,

$$\lambda = \frac{k_\perp v_\perp}{\omega_B}\,, \quad \omega_B = \frac{e\,B_0}{m_0 c}\sqrt{1-\beta^2}\,. \tag{15}$$

В той же системе координат вектор $\mathbf{a}$ имеет вид ($\cos\theta = k_\parallel/k$):

$$\mathbf{a} = a_\gamma\,\{\mathrm{i}\alpha_{x_0},\, 1,\, \mathrm{i}\alpha_{z_0}\}$$

$$a_\gamma{}^2 = \frac{1}{1+\alpha_x{}^2}\,, \quad \alpha_x = \alpha_{x_0}\cos\theta - \alpha_{z_0}\sin\theta\,. \tag{16}$$

Для холодной плазмы, когда тензор $\varepsilon_{\alpha\beta}$ имеет компоненты

$$\varepsilon_{x_0x_0} = \varepsilon_{y_0y_0} \equiv \varepsilon\,, \quad \varepsilon_{z_0z_0} \equiv \eta\,, \quad \varepsilon_{x_0y_0} = -\varepsilon_{y_0x_0} \equiv \mathrm{i}g\,,$$

$$\varepsilon_{x_0z_0} = \varepsilon_{y_0z_0} = \varepsilon_{z_0x_0} = \varepsilon_{z_0y_0} = 0\,, \tag{16a}$$

величины α_{x_0}, α_{z_0} и N^2 определяются выражениями

$$\alpha_{x_0} = \frac{N^2-\varepsilon}{g}\,, \quad \alpha_{z_0} = -\frac{N^2 g\sin\theta\cos\theta}{N^2(\varepsilon\sin^2\theta + \eta\cos^2\theta) - \varepsilon\eta}\,, \tag{17}$$

$$N^2 = \frac{2(\varepsilon^2-g^2)}{2\varepsilon - \left(\varepsilon - \dfrac{\varepsilon^2-g^2}{\eta}\right)\sin^2\theta \pm \sqrt{\left(\varepsilon - \dfrac{\varepsilon^2-g^2}{\eta}\right)^2\sin^4\theta + 4g^2\cos^2\theta}}\,. \tag{18}$$

Вычисляя входящую в (11a) свертку $\Pi_{\beta\alpha}{}^{(m)}\, a_\alpha\, a_\beta{}^*$, находим

$$\Pi^{(m)}_{\beta\alpha} a_\alpha a^*_\beta = \frac{1}{1+a_x{}^2}\left\{\left(\frac{m v_\perp}{\lambda}\,\alpha_{x_0} + v_\parallel \alpha_{z_0}\right) J_m(\lambda) + v_\perp J_m{}'(\lambda)\right\}^2. \tag{19}$$

В сумме, определяющей значение $G_{\beta\alpha}{}^1$ целесообразно при суммировании по отрицательным m заменить m на $-m$. Выполняя в (11a) интегрирование по k с помощью δ-функций, входящих в $L_{\alpha\beta}{}'$ (12), получим выражение для потерь на излучение в следующем виде

$$-\frac{\mathrm{d}E}{\mathrm{d}t} = \sum_{l=1}^{2}\int\limits_{-1}^{1}\mathrm{d}\mu\int\limits_0^\infty \mathrm{d}\omega\,\frac{e^2\omega^2}{c^3}\,\frac{N_l}{1+a^2{}_{xl}}\left\{I_0 + \sum_{m=1}^{\infty}(I_m + I_{-m})\right\}, \tag{20}$$

$$I_0 = (\alpha_{z_0} v_\parallel J_0 + v_\perp \mathrm{J}_0{}')^2\,\delta(\omega - \omega\beta_\parallel \mathrm{N}_l \mu)\,,$$

$$I_m = \left[\left(\alpha_{x_0}\frac{m v_\perp}{\lambda_l} + \alpha_{z_0} v_\parallel\right) J_m + v_\perp J_m{}'\right]^2 \times \delta(\omega - m\omega_\beta - \omega\beta_\parallel N_l \mu)\,.$$

$$I_{-m} = \left[\left(\alpha_{x_0}\frac{m v_\perp}{\lambda_l} - \alpha_{z_0} v_\parallel\right) J_m - v_\perp J'{}_m\right]^2 \times \delta(\omega + m\omega_\beta - \omega\beta_\parallel N_l \mu)\,.$$

Здесь $\beta_\parallel = v_\parallel/c$, аргументом функций Бесселя является $\lambda_l = (\omega/\omega_B)\,(v_\perp/c)\, N_l\sqrt{1-\mu^2}$. Аргументы δ-функций показывают, что I_0 определяет черенковское излучение заряда, I_m — циклотронное излучение с учетом нормального допплер-эффекта (при $\omega_B > 0$), I_{-m} — излучение с учетом аномального допплер-эффекта [11]. Мы будем в дальнейшем интересоваться излучением только на резонансных частотах $\omega = m\,\omega_B$, которое определяется слагаемым с I_m.

3. Излучение ионов в холодной плазме на резонансных частотах

Величина показателя преломления при $\omega \sim m\,\omega_{Bi}$ ($m = 1, 2, \ldots$) зависит от значения параметра $A = \omega_{0i}/\omega_{Bi}$, равного отношению ленгмюровской ионной частоты к ионной циклотронной частоте, или, что то же самое, отношению скорости

света к альфвеновской скорости. Для водородной плазмы этот параметр равен

$$A = \frac{0{,}14\sqrt{n_0}}{B_0}\,. \tag{21}$$

Будем предполагать, что $A \gg 1$. При $B_0 = 3\cdot 10^3$ гаусс (обычно используемое поле в Огре) это условие выполняется при $n_0 \gg 5\cdot 10^8$ см$^{-3}$. Заметим, что на Огре в тех режимах, где существенно проявляются резонансные пики напряженности электрического поля на обертонах ($m \sim 3$-4), плотность холодной плазмы превосходит значение $n_0 = 10^9$ см$^{-3}$. В более плотной плазме ($n_0 \gtrsim 10^{13}$) условие $A \gg 1$ заведомо выполняется при любых реально достижимых значениях B_0.

При условии $A \gg 1$ величины ε, g, η, определяющие показатель преломления и вектор поляризации $\mathbf{a}$, равны ($x \equiv \omega/\omega_{\mathrm{Bi}}$)

$$\varepsilon = \frac{A^2}{1-x^2}\,, \quad g = \frac{A^2 x}{1-x^2}\,, \quad \eta = -\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}}\frac{A^2}{x^2}\,. \tag{22}$$

(m_{i}, m_{e} — массы иона и электрона, соответственно).

Большая величина компоненты тензора электрической проницаемости $\varepsilon_{z_0 z_0} = \eta$ означает, что продольное (по отношению к $\mathbf{B}_0$) электрическое поле $E_{z_0} = D_{z_0}/\eta$ мало (оно уничтожается движением электронов вдоль магнитных силовых линий). Как видно из (17) и (22) $(\alpha_{z_0}/\alpha_{x_0}) \sim m_{\mathrm{e}}/m_{\mathrm{i}} \ll 1$. Опустим поэтому слагаемое с α_{z_0} в (20). Производя интегрирование по ω, получим для интенсивности излучения P_m на резонансных частотах (слагаемое с I_m в (20)) для каждой из двух поляризаций:

$$P_m = \frac{e^2\,\omega^2{}_{\mathrm{Bi}}}{c^3}\,v_\perp{}^2\,A\cdot p_m\,, \tag{23}$$

$$p_{\mathrm{m}} = \int\limits_{-1}^{1} \frac{x^2 n}{1+\alpha^2\mu^2}\,\frac{\left[\alpha\,\frac{m}{\lambda}\,J_m(\lambda) + J_m{}'(\lambda)\right]^2}{|F|}\,\mathrm{d}\mu\,, \tag{24}$$

$$F = 1 - \beta_2\mu\,\frac{\partial\,(x\,n)}{\partial x}\,, \tag{25}$$

$$\alpha \equiv \alpha_{x_0} = \frac{n^2\,(1-x^2)-1}{x}\,. \tag{26}$$

Здесь $\lambda = \beta_1\,x\,n\,\sqrt{1-\mu^2}$; подинтегральное выражение в (24) следует брать при значении x, удовлетворяющем уравнению (нуль аргумента δ-функции, входящей в I_m)

$$x = \frac{m}{1-\beta_2\cdot\mu\cdot n\,(x,\mu)} \tag{28}$$

В формулах (24—28) введены обозначения

$$\beta_1 = \frac{v_\perp}{c}\,A = \frac{v_\perp}{c_{\mathrm{A}}}\,,\ \beta_2 = \frac{v_\parallel}{c}\,A = \frac{v_\parallel}{c_{\mathrm{A}}}\,,\ n = \frac{N}{A}\,. \tag{29}$$

Для величины n получаем из (18) с учетом (22)

$$n_\pm^2 = \frac{2}{1+\mu^2 - \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}x^2(1-\mu^2) \pm \sqrt{(1-\mu^2)^2\left(1+\frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}x^2\right)^2 + 4x^2\mu^2}} \tag{30}$$

Зависимость квадрата показателя преломления от частоты изображена на рис. 1. Как видно,

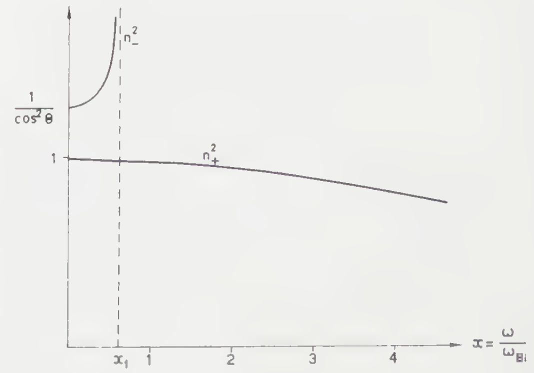

Рис. 1

$n_+{}^2 > 0$ во всей рассматриваемой области частот, тогда как $n_-{}^2 > 0$ лишь при $x < x_1$, где

$$x_1 = \frac{\mu}{\sqrt{\mu^2 + \frac{m_{\mathrm{e}}}{m_{\mathrm{i}}}(1-\mu^2)}}\,. \tag{31}$$

В выражении для $n_+{}^2$ слагаемые с m_e/m_i можно опустить. Для $n_-{}^2$ это можно сделать лишь для углов вне узкого конуса $\mu \gtrsim (m_e/m_i)^{1/2}$. Упрощенное выражение для $n_\pm^2$ имеет вид [12]

$$n_\pm^2 = \frac{2}{1+\mu^2 \pm \sqrt{(1-\mu^2)^2 + 4x^2\mu^2}}\,. \tag{32}$$

Рассмотрим сначала излучение волн, характеризующихся показателем преломления $N_+ = A n_+$. Функция F в этом случае равна

$$F = 1 - \beta_2\mu\,n_+\left(1 - \frac{n_+{}^2\,\mu^2 x^2}{\sqrt{1+2\,(2x^2-1)\,\mu^2+\mu^4}}\right). \tag{33}$$

Величина p_m была определена численным интегрированием при различных значениях параметров β_1, β_2 и m. Результаты приведены в таблице I. Из таблицы видно, как с увеличением $\beta_1 = v_\perp/c_A$ максимум интенсивности излучения смещается в сторону больших m*.

Обращает на себя внимание тот факт, что при малых $\beta_2 = v_\parallel/c_A$ интенсивность излучения на циклотронной частоте значительно меньше, чем интенсивность излучения на соседних обертонах. Чтобы выяснить причину этого различия рассмотрим формулы для излучения при $\beta_2 = 0$ в предположении $\lambda \ll 1$. Из (28) следует, что $x = m$, так что

* Заметим, что как следует из (23), интенсивность излучения иона может сравниваться с интенсивностью дипольного излучения электрона в вакууме при $Ap_m \sim (m_i/m_e)^2$. Как видно из таблицы при $\beta_1 \sim 1$-$1{,}5$ и $m > 20$, p_m достигает значений $\gtrsim 10^2$. Так что при $A > 10^3$ суммарная интенсивность излучения иона может оказаться больше интенсивности дипольного излучения электрона, движущегося с той же скоростью в вакууме.

ТАБЛИЦА I. *Значения* p_m.

m		1	2	3	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42
β_1	β_2																							
0,1	0,01	$2{,}6\cdot10^{-6}$	$4{,}2\cdot10^{-3}$	$3{,}1\cdot10^{-4}$																				
	0,1	$1{,}4\cdot10^{-4}$	$4{,}3\cdot10^{-3}$	$3{,}1\cdot10^{-4}$																				
0,5	0,05	$7{,}6\cdot10^{-4}$	0,11	0,17	0,14	$5{,}8\cdot10^{-2}$	$1{,}7\cdot10^{-2}$																	
	0,5	$7{,}1\cdot10^{-3}$	0,13	0,18	0,15	$6\cdot10^{-2}$	$1{,}7\cdot10^{-2}$																	
0,7	0,07	$2{,}8\cdot10^{-3}$	0,21	0,55	0,8	0,95	0,81	0,59	0,40	0,25														
	0,7	$2{,}3\cdot10^{-2}$	0,29	0,65	0,9	1,03	0,86	0,62	0,41	0,26														
0,8	0,08	$4{,}6\cdot10^{-3}$	0,28	0,84	1,5	2,5	3,1	3,2	3,1	2,8	2,4	2,0	1,67	1,33	1,03	0,78	0,6							
	0,8	$3{,}8\cdot10^{-2}$	0,4	1,04	1,7	2,8	3,3	3,4	3,2	2,9	2,5	2,1	1,7	1,36										
0,9	0,09	7.10^{-3}	0,35	1,2	2,4	5,4	8,7	12	15	17	19	21	22	22,9	23,1	22,8	22	21	20					
1,0	0,1	$1{,}1\cdot10^{-2}$	0,43	1,6	3,5	9,6	18	30	44	61	81	103	128	156	187	220	255	294						
	1,0	$9{,}6\cdot10^{-2}$	0,75	2,2	4,3	11	20	32	46	64	83	106	131	158										
1,3	0,13	$2{,}8\cdot10^{-2}$	0,7	2,8	6,6	20	41	65	88	108	125	143	168	208	267	345	436	527	608	668	707	731	755	797
1,5	0,15	$4{,}6\cdot10^{-2}$	0,87	3,1	6,8	16	22	29	46	75	105	122	131	152	202	265	309	322	333	382	475			
3,0	0,3	0,26	0,46	1,0	1,2	2,3	4,0	6,0					24											

$$p_m = \int_1^1 \frac{m^2 n}{1+\alpha^2\mu^2}\cdot\frac{(1+\alpha)^2\lambda^{2(m-1)}}{2^{2m}[(m-1)!]^2}\,\mathrm{d}\mu \tag{34}$$

$$\alpha = \frac{n^2(1-m^2)-1}{m},$$

$$n^2 = \frac{2}{1+\mu^2+\sqrt{1+2(2m^2-1)\mu^2+\mu^4}}, \tag{35}$$

$$\lambda = \beta_1 m n \sqrt{1-\mu^2}.$$

При $m=1$, как видно, $\alpha=-1$ и, следовательно, $p_1=0$. Таким образом, в дипольном приближении излучение волн рассматриваемой поляризации на циклотронной частоте отсутствует. В случае излучения электронов такой же результат был получен в работах [5], [6]. Условие $\alpha=-1$ $(\alpha_{x_0}=-1)$ означает, что проекция электрического вектора на плоскость, перпендикулярную к постоянному магнитному полю, вращается по кругу и при этом в направлении, противоположном вращению заряда. Отсутствие резонанса между зарядом и полем и является причиной отсутствия излучения.

Заметим, что при больших значениях m в интеграле (24) основной вклад дают малые значения $\mu \lesssim 1/m$. В этой области углов $n\approx 1$, так что соответствующая фазовая скорость волн близка к альфвеновской скорости $V\approx c_A$. Формула для показателя преломления (32) пригодна лишь для частот, не превышающих нижнюю гибридную частоту [13] $\omega=\omega_{\mathrm{Bi}}\cdot A\left(\frac{m_{\mathrm{i}}/m_{\mathrm{e}}}{m_{\mathrm{i}}/m_{\mathrm{e}}+A^2}\right)^{1/2}$, так что значения m не должны превышать меньшую из следующих двух величин: $(m_{\mathrm{i}}/m_{\mathrm{e}})^{1/2}$, A.

Обратимся теперь к волнам с другой поляризацией.

Волны с показателем преломления $N_-=An_-$ могут излучаться лишь при $x<x_1\lesssim 1$ (31). При $\beta_2=0$ (движение по окружности), согласно (28) $x=1$ $(\omega=\omega_{\mathrm{Bi}})$ и, следовательно, излучение отсутствует. При $\beta_2\neq 0$ (движение по винтовой линии) резонансная частота смещается при $\beta_2\mu<0$ вследствие допплер-эффекта в сторону более низких частот и, следовательно, излучение появляется. Оценим его интенсивность в предположении $|\beta_2\mu n|\ll 1$. Полагая, согласно (28), $x=1+\beta_2\mu n_-$, получаем из (30) приближенно

$$n_-^{\,2} = -\frac{1+\mu^2}{2\beta_2 n_-\mu^3+\dfrac{m_{\mathrm{e}}}{m_{\mathrm{i}}}(1-\mu^2)}. \tag{36}$$

При $1\geqslant\mu^2>(m_e/m_i)(2\beta_2)^{-2/3}\equiv\mu_0^{\,2}$ можно положить $n_-^{\,2}=-(2\beta_2 n_-\mu^3)^{-1}$, т.е. $|n_-\beta_2\mu|^3=\beta_2^{\,2}/2$ (так что условие $|\beta_2\mu n_-|\ll 1$ сводится к $\beta_2\ll 1$). Используя это значение n_-, получим

$$p_1 \approx \frac{1}{6\sqrt[3]{2\beta_2}}\int_1^{\mu_0}\frac{\mathrm{d}\mu}{\mu^3} \sim \sqrt[3]{\beta_2}\,\frac{m_{\mathrm{i}}}{m_{\mathrm{e}}}. \tag{37}$$

Благодаря множителю m_i/m_e интенсивность излучения оказывается (даже при небольших β_2) очень большой. Этот результат является, однако, следствием идеализации исходных условий, приводящих к возможности как угодно больших значений N_-. Нетрудно видеть, что все излучение приходится на частоты, весьма близкие к частоте $\omega=\omega_{\mathrm{Bi}}\,x_1$ при которой $N_-^{\,2}=\infty$. Значение $N_-^{\,2}$ в области излучаемых частот оказывается поэтому весьма большим $N_-^{\,2}\approx A^2(m_{\mathrm{i}}/m_{\mathrm{e}})$. Поэтому при описании реальной плазмы следует учитывать факторы, приводящие к ограничению значений $N_-^{\,2}$. Такими факторами могут явиться тепловое движение зарядов холодной плазмы, их столкновения, неоднородность магнитного поля. Отсюда следует, что при сравнении экспериментальных данных по измерению резонансных пиков напряженности электрического поля с интенсивностью излучения, рассчитанной в настоящей работе, следует исключать из рассмотрения циклотронную частоту.

Если учитывать только обертоны $(\omega=m\,\omega_{\mathrm{Bi}}$, $m\geqslant 2)$, то смещение максимума интенсивности излучения при увеличении параметра $\beta_1=(v_\perp/c)A$ находится в качественном согласии со смещением наблюдаемых [1] резонансных пиков напряженности электрического поля при увеличении плотности холодной плазмы (напомним, что $A\sim\sqrt{n_0}$).

4. Обсуждение результатов

Приведенные выше расчеты показывают, что интенсивность излучения быстрых ионов в холодной плазме растет с увеличением плотности последней. Характер распределения интенсивности излучения по гармоникам также существенно зависит от плотности плазмы; при увеличении плотности происходит сдвиг максимума интенсивности излучения в сторону высоких обертонов. Оба эти обстоятельства, повидимому, можно использовать в диагностических целях для определения плотности плазмы.

Следует, однако, иметь ввиду, что приведенные формулы определяют интенсивность излучения на больших расстояниях R по сравнению с излучаемой длиной волны λ. В лабораторных условиях требование $R \gg \lambda$ выполняется (если антенна находится в плазме) при сравнительно большой плотности плазмы, а именно при

$$\nu = \frac{e^2 N_i}{m_i c^2} \gg \frac{\pi^2}{m^2}$$

($N_i = n_0 \cdot \pi a^2$ — „погонное число ионов" в столбе плазмы радиусом a, m — номер излучаемой гармоники).

В заключение авторы выражают глубокую благодарность А. Е. БАЖАНОВОЙ за проведение численных расчетов.

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ИЗМЕРЕНИЕ ЭЛЕКТРОННОЙ ТЕМПЕРАТУРЫ ПЛАЗМЫ В МОЩНОЙ УДАРНОЙ ВОЛНЕ

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ОРДЕНА ЛЕНИНА ИНСТИТУТ АТОМНОЙ ЭНЕРГИИ ИМ. И. В. КУРЧАТОВА АКАДЕМИИ НАУК СССР, МОСКВА, СССР

Методом вытесненного магнитного потока измерена электронная проводимость дейтериевой плазмы за сильной ударной волной. При скоростях ударной волны от $0{,}9 \cdot 10^7$ до $1{,}25 \cdot 10^7$ см/сек найдена электронная температура в 50 и 90 eV. Проведено сравнение полученной электронной температуры с рассчитанной по скорости ударной волны. Пьезоэлектрические измерения и сжатие плазмы сильным магнитным полем дали одно и то же значение газокинетического давления плазмы.

Введение

Исследования, проводимые с целью получения управляемых термоядерных реакций, показали, что импульсный разряд может служить источником сильных ударных волн [1].

Одним из вариантов создания сильных ударных волн является импульсный разряд в цилиндрической камере. По оси камеры образуется плазменный шнур, в котором плазма находится под большим давлением из-за пинч-эффекта. Если в электроде камеры сделать отверстие по оси и в отверстие поставить цилиндрическую трубку, то в момент максимального сжатия пинч будет действовать как поршень, перед которым образуется ударная волна, распространяющаяся по трубке с большой скоростью. Определив параметры плазмы за ударной волной, можно сделать некоторые выводы относительно состояния плазмы в цилиндрической камере.

В [1, 2, 3, 4] изучалось поведение газа, нагретого ударной волной, с помощью импульсного разряда. Однако, во всех указанных работах существенным недостатком является присутствие в плазме, движущейся за ударной волной, магнитного поля, связанного с основным разрядом.

В данной работе магнитное поле основного разряда не проникало в трубку, по которой распространялась ударная волна. На трубку наматывались витки, определяющие производную магнитного поля. Предусматривалось последующее интегрирование сигнала на усилителе осциллографа. Измерения показали, что импульс магнитного поля очень мал и быстро затухает по мере распространения ударной волны (за время 0,5 μsec). Наличие такого слабого импульса магнитной компоненты можно объяснить как результат некоего неравновесного процесса, происходящего за ударной волной сразу после образования ударной волны.

Отметим, что в работе [5] магнитное поле, связанное с основным током разряда, имеет большую величину и распространяется вместе с потоком плазмы, движущейся за фронтом ударной вольны. В описываемой работе геометрия эксперимента исключала проникновение основного поля в трубку.

1. Определение электронной температуры T_e плазмы за ударной волной методом вытесненного магнитного потока

1.1. Теоретические предпосылки

Плазма, движущаяся за ударной волной в постоянном продольном диагностическом магнитном поле, аналогична проводнику с проводимостью σ, определяемой электронной температурой T_e.

Решив задачу о проникновении магнитного поля внутрь плазмы, найдем ее проводимость в предположении, что:

1. $\sigma = \text{const.}$ по сечению и длине (плазма имеет форму несжимаемого цилиндра, радиус которого меньше, но может быть сравним с толщиной скин-слоя).
2. $\sigma = \text{const.}$ по времени.
3. Время влета плазмы в магнитное поле много меньше, чем время диффузии поля внутрь плазмы.

Если перейти в систему координат, движущуюся со скоростью, равной скорости v_z плазмы за ударной волной, то задача сводится к следующей. В момент времени t_0 включается продольное поле H_0, которое проникает внутрь неподвижной плазмы. Тогда из уравнений:

$$\operatorname{rot} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} \tag{1}$$

$$\operatorname{rot} \mathbf{H} = \frac{4\pi}{c}\mathbf{j} \tag{2}$$

$$\mathbf{E} = \frac{1}{\sigma}\mathbf{j} \tag{3}$$

следует, что

$$\Delta \mathrm{H} = \frac{4\pi\sigma}{c^2}\frac{\partial \mathbf{H}}{\partial t} \tag{4}$$

Введем функцию $F(r, t)$ такую, что

$$H(r,t) = F(r,t) + H_0 \tag{5}$$

где H_0 — поле, в которое влетает плазма.

Тогда (4) запишется:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) = \frac{4\pi\sigma}{c^2}\frac{\partial F}{\partial t} \tag{6}$$

Это уравнение теплопроводности с граничным условием:

$$F(r_0, t) = 0 \tag{7}$$

где r_0 — радиус плазмы и с начальным условием:

$$F(r, 0) = -H_0 \tag{8}$$

Решение имеет следующий вид:

$$H = H_0 - 2H_0 \sum_{n=1}^{\infty} \frac{J_0(\mu_n r/r_0)}{\mu_n \cdot J_1(\mu_n)} \cdot \exp\left(-\frac{\mu_n^2 c^2}{4\pi\sigma r_0^2} \cdot t\right) \tag{9}$$

где μ_n — корни уравнения Бесселя:

$$J_0(\mu_n) = 0 \tag{10}$$

Отсюда находим поток магнитного поля, проинтегрировав его напряженность по сечению плазмы

$$\varphi = \int_S H\,\mathrm{d}S = H_0 \pi r^2 - 4H_0 \pi r_0^2 \sum_{n=1}^{\infty} \frac{\exp[-(\mu_n^2 c^2/4\pi\sigma r_0^2)t]}{\mu_n^2} \tag{11}$$

Это же равенство можно записать следующим образом:

$$\varphi = \varphi_0 - \varphi_t \tag{12}$$

где
$$\varphi_0 = H_0 \pi r_0^2 \tag{13}$$

магнитный поток через сечение, равное сечению плазмы до ее впуска в поле $H_0 \cdot \varphi_t$ — разница между φ и φ_0, т.е. тот поток, который вытесняется из пространства при введении в него плазмы за время t. Теперь можно построить семейство кривых

$$\frac{\varphi_t}{\varphi_0} = 4 \sum_{n=1}^{\infty} \frac{\exp[-(\mu_n^2 c^2/4\pi r\sigma_0^2)t]}{\mu_n^2} \tag{14}$$

как функцию времени, меняя параметр σr_0^2. Кривые для экспериментального $r_0 = 0{,}55$ см построены на рис. 1. Зная отношения φ_t/φ_0, можно

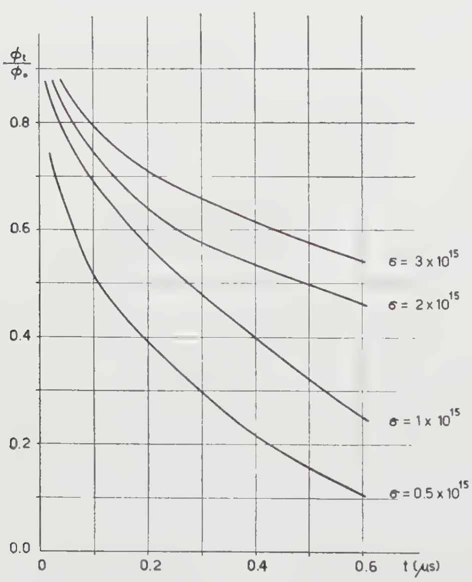

Рис. 1 Зависимость φ_t/φ_0 от времени проникновения магнитного поля.
В качестве параметра взято σr_0^2, где σ — проводимость, r_0 — начальный радиус плазмы. Измерения проводились в фиксированные моменты t_1 и t_2 от начала вхождения волны в область с диагностическим магнитным полем.

легко найти проводимость σ для конкретного случая. Отметим, что в эксперименте r_0 определялось из дополнительных измерений по четырем контрольным катушкам.

1.2. ЭКСПЕРИМЕНТ

Использовалась цилиндрическая разрядная камера (рис. 2). Батарея конденсаторов в 85 μF

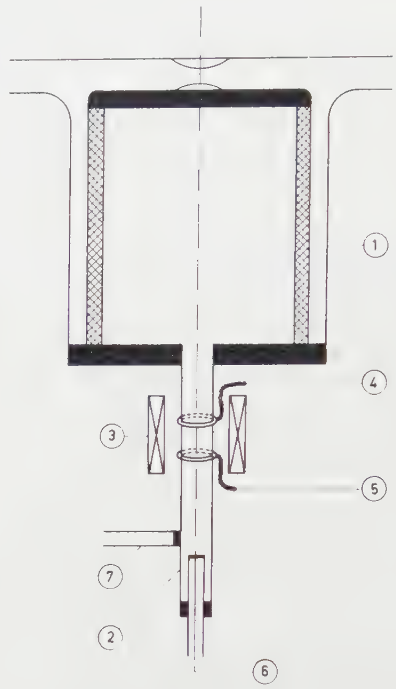

Рис. 2 Схема экспериментальной установки.
1 — разрядная камера, диаметр 40 см и расстояние между электродами 45 см.
2 — стеклянная трубка,
3 — катушка для создания продольного магнитного поля H_0,
4, 5 — измерительные витки,
6, 7 — пьезокерамические датчики давления.

заряжалась до 30 kV, скорость нарастания тока $4 \cdot 10^{11}$ A/сек, рабочий газ дейтерий, начальные давления 0,2 и 0,05 мм рт. ст.

В работе [6] проводимость сильно ионизованного аргона, созданного в ударной трубе, измерялась с помощью магнитной измерительной катушки. Калибровка сигналов производилась простреливанием через магнитную измерительную катушку алюминиевых образцов известной проводимости. Примененный метод измерения пригоден для относительно низких параметров плазмы, когда время диффузии магнитного поля мало по сравнению со временем влета плазмы в магнитное поле.

В описываемых экспериментах диагностическое магнитное поле создается катушкой, намотанной медной шинкой 2×1 мм в 9 слоев. На концах ее сделаны дополнительные обмотки для обеспечения формы магнитного поля, близкой к прямоугольной. С той же целью на концах катушки сделаны железные щечки. Шинка наматывалась на медную трубочку, которая служила экраном, препятству-

ющим распространению магнитного потока, вытесняемого плазмой. Магнитная катушка питалась от батареи аккумуляторов, дающей 12, 24 и 48 V. При этом поля внутри катушки равнялись соответственно 1150, 2300 и 4600 ое. Вытесненный плазмой поток измерялся дифференцирующими витками, находящимися в катушке, создающей диагностическое магнитное поле. В зависимости от эксперимента число витков изменялось. Сигнал с витков

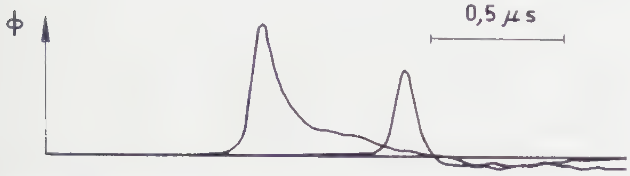

Рис. 3 Осциллограмма проинтегрированных сигналов с измерительных витков 4, 5.

(рис. 3) регистрировался на двухлучевом импульсом осциллографе ОК-21 после интегрирования и усиления.

По осциллограммам вытесненного потока находилась скорость ударной волны по времени прохождения ее между двумя измерительными витками. Она лежала в пределах от $0{,}9 \cdot 10^7$ см/сек до $1{,}25 \cdot 10^7$ см/сек. Замечено, что скорость ударной волны на протяжении всей трубки ($l = 50$ см) меняется мало. Поэтому для дальнейших расчетов ее можно считать между двумя измерительными витками постоянной и равной 10^7 см/сек.

Расчет проводимости проводился не только в слабом постоянном, но и сильном диагностическом магнитном поле и дал значение $\sigma = (5 \pm 2) \cdot 10^{15}$ ед. CGSE. Соответствующая этой проводимости электронная температура, следуя [7], вычисляется по формуле:

$$T_e = \left(\frac{\sigma}{1{,}3} \cdot 10^{-13}\right)^{\frac{2}{3}} \tag{15}$$

T_e — электронная температура в eV, тогда $T_e \cong 50$ eV для $\sigma = 5 \cdot 10^{15}$ ед. CGSE. Максимальное наблюдавшееся значение $\sigma \cong 10^{16}$ ед. CGSE, соответствующая ей электронная температура $T_e \cong 90$ eV.

2. Обсуждение результатов

Применив к движению ударного фронта законы сохранения массы, импульса и энергии можно оценить температуру плазмы за ударной волной, зная только один параметр, например, скорость ударной волны. Простые выкладки показывают, что температура плазмы, рассчитанная при услови термодинамического равновесия за ударной волной, отличается не сильно от электронной температуры, полученной методом вытесненного магнитного потока и равняется, примерно, 40 eV.

Проделан ряд экспериментов по удержанию плазмы, движущейся за ударной волной сильным магнитным полем. Найдено, что плазма начинает сжиматься при напряженности магнитного поля $H \cong 10$ koe, что соответствует давлению магнитного поля $H^2/8\pi = 4$ атм, следовательно, газокинетическое давление внутри плазмы, за ударной волной, $\cong 4$ атм. Из закона сохранения импульса на ударной волне находим давление p_2 за ударной волной по ее скорости θ,

$$p_2 = \varrho_1 \cdot \theta^2 \cdot \left(1 - \frac{\varrho_1}{\varrho_2}\right), \tag{16}$$

ϱ_1/ϱ_2 — отношение плотностей газа перед и за ударной волной, это отношение в условиях полной ионизации равно 0,25. Для первоначального давления $p_1 = 0{,}2$ мм рт. ст. и $\theta = 10^7$ см/сек вычисленное значение p_2 оказывается равным 3,5 атм.

Многочисленные пьезоизмерения (пьезодатчики ставились в торец и заподлицо со стенкой трубки на различных расстояниях) дали значения $p \cong 4$ атм. Все давления p_2, полученнные различными способами, отличаются не сильно, а ошибки лежат в пределах точности эксперимента.

Форма и время пьезоимпульса (рис. 4), а также предварительные расчеты, показывающие, что

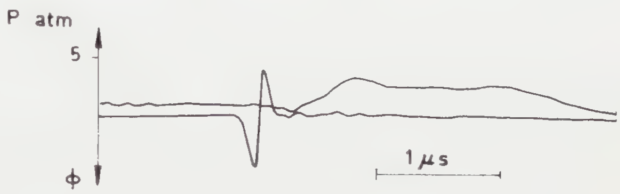

Рис. 4 Осциллограмма производной магнитного потока в сечении витка 5 и сигнала пьезокерамического датчика 6. Поверхность датчика на 1 см ниже плоскости витка 5.

термодинамическое равновесие за ударной волной наступает весьма быстро (за время порядка 10^{-7} сек) открывают заманчивые перспективы для экспериментаторов, работающих в области физической газовой динамики больших скоростей.

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LOSS OF PARTICLES IN A PINCHED DISCHARGE IN AN AXIAL MAGNETIC FIELD*

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A calculation of the loss of deuterons from a pinched current to the wall of the container has been given by G. P. Thomson [*Phil. Mag.* **32** (1958) 886]. This calculation is extended so that account is taken of an axial magnetic field. It is found that such a field can materially reduce the particle loss.

1. Introduction

If a strong current is passed through a low density gas in a cylindrical container, the self attraction of the parallel currents carried by the electrons causes an electron concentration near the axis of the cylindrical discharge and a radial electric field is set up between the axis of the discharge and its containing surface. Such a configuration is, however, found to be unstable. The forces arising from the twisting of an axial current produce further bending of the discharge column and the constricted matter comes in contact with the wall of the vessel. An axial magnetic field is usually applied to stabilize such currents as occur in Zeta, Scepter, Stellerator and other fusion machines. However, it is found that the axial magnetic field stabilizes the pinch only for certain small wavelength disturbances [1, 2, 3].

Thomson [4] studied the problem of the loss of heat from the pinch due to the collision of the particles with the wall. He found that the loss of heat is reduced by the binding of the paths of both nuclei and electrons by the toroidal magnetic field of the current, as well as by the electrostatic potential created by the concentration of electrons near the axis of the cylindrical discharge. It is to be noted that the electric field, the particle density and the magnetic field (outside the pinched region) decrease towards the wall.

The object of the present paper is to estimate how the loss of particles to the wall is affected by the presence of an imposed magnetic field parallel to the axis of the discharge.

2. Equilibrium configuration

Assuming that the velocity of the electrons is wholly axial in the steady state configuration and that the velocity of the deuterons is negligible compared to that of the electrons, we have the following equation for the radial equilibrium of the deuterons:

$$n e E_r = k T_{\mathrm{d}} \frac{\mathrm{d}n}{\mathrm{d}r}, \tag{1}$$

and for the equilibrium of the electrons

$$-n e E_r - n e v_{\mathrm{e}} H = k T_{\mathrm{e}} \frac{\mathrm{d}n}{\mathrm{d}r}. \tag{2}$$

Here n is the particle density of the electrons as well as deuterons, v_{e} is the mean axial velocity of the electrons; H is the toroidal magnetic field in gauss; E_r is the radial electric field in e.m.u. and is negative, T_{d} and T_{e} are the temperatures for deuterons and electrons, respectively. The presence of the axial magnetic field does not change the steady state described by Thomson.

If $T_{\mathrm{d}} = T_{\mathrm{e}} = T$, Eq. (1) can be written:

$$E_r = \frac{kT}{e} \frac{\mathrm{d}}{\mathrm{d}r} (\ln n), \tag{1a}$$

which integrates to

$$n = n_0 \exp(-eV/kT),$$
$$V = -\int_0^r E_r \mathrm{d}r. \tag{3}$$

Eq. (2) yields

$$v e H = E_r - \frac{kT}{e} \frac{\mathrm{d}}{\mathrm{d}r} (\ln n) = -2 E_r. \tag{4}$$

Thus the result that the pinch force on the moving electrons is twice the radial electric field still holds when the axial magnetic field is present.

The equilibrium radius of the pinch will be written r_0. Following Thomson we shall assume that in the region outside the pinch V is of the form $2\sigma \log(r/r_0)$, i.e. the electrostatic potential due to a uniformly charged cylinder of radius r_0, when the region outside the pinch is electrically neutral. Our calculations follow the same stages as those of Thomson.

3. Equation of motion for a deuteron and its solution

We calculate the chance that a deuteron of mass m and charge $+e$ which has made a collision at r, θ, z (in cylindrical coordinates) will hit the wall. The equation of motion of the deuteron, in e.m.u., is

$$m \frac{\mathrm{d}^2 \mathbf{r}}{\mathrm{d}t^2} = e(\mathbf{E} + \mathbf{v} \times \mathbf{H}). \tag{5}$$

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In our case this yields three equations

$$m\,(\ddot r - r\dot\theta^2) = e\,E_r + e\,H_1\,r\,\dot\theta + e\,H\,\dot z\,, \tag{6}$$

$$\frac{m}{r}\,\frac{\mathrm{d}}{\mathrm{d}t}\,(r^2\,\dot\theta) = -eH_1\,\dot r\,, \tag{7}$$

$$m\,\ddot z = -\,e\,H\,\dot r\,. \tag{8}$$

Here $H = (2\,I/r)$ with I the total current strength in the pinch. H_1 is the uniform axial magnetic field applied externally.

Eq. (8) integrates to

$$\dot z = \dot z_0 - \frac{2\,I\,e}{m}\,\ln\frac{r'}{r}\,, \tag{8a}$$

where $\dot z_0$ is the initial axial velocity of the deuteron, r its initial radius and r' its final radius.

Eq. (7) integrates to

$$r\,\dot\theta = \frac{C}{r} - \frac{e\,H_1}{2\,m}\,r\,. \tag{7a}$$

where C is the constant of integration.

At a point where r' has its maximum we have $\dot r = 0$. The condition for the collision with the wall is

$$r'_{\max} \geqslant R\,, \tag{9}$$

R being the radius of the wall. At such a point the equation of energy gives

$$\frac{1}{2}\,m\left[\dot z^2 + \left(\frac{C}{r'} - \frac{e\,H_1}{2\,m}\,r'\right)^2\right]$$
$$= \frac{1}{2}\,m\,\omega^2 - 2\,\sigma\,e\,\ln\frac{r'}{r} + \frac{1}{2}\,m\left(\frac{C}{r} - \frac{e\,H_1}{2\,m}\,r\right)^2. \tag{10}$$

where we have written $\frac{1}{2}\,m\,\omega^2$ for the initial kinetic energy of the particle in so far as it corresponds to its degrees of freedom in the axial plane. The equation can be written

$$\left[\dot z_0 - \frac{2\,I\,e}{m}\,\ln\frac{r'}{r}\right]^2 = \omega^2 - \frac{4\,\sigma\,e}{m}\,\ln\frac{r'}{r}$$
$$+ \frac{e^2\,H_1^{\,2}}{4m^2}\,(r^2 - r'^2) + C^2\left(\frac{1}{r^2} - \frac{1}{r'^2}\right). \tag{11}$$

Following Thomson we neglect $C^2\,(1/r^2 - 1/r'^2)$, the error in neglecting this term being smaller than the error involved in taking ω as corresponding to two or three degrees of freedom. With these assumptions Eq. (11) yields:

$$\dot z_0 = a\ln\frac{r'}{r} \pm \left[\omega^2 - b\ln\frac{r'}{r} - p\,(r'^2 - r^2)\right]^{\frac{1}{2}}, \tag{11a}$$

where

$$a = 2\,I\,e/m\,,\quad b = 4\,\sigma\,e/m\,,\quad p = e\,H_1/2\,m\,.$$

The condition for the particle to hit the wall is that Eq. (11a) shall give a real value of $\dot z_0$ for $r' \geqslant R$ and $r < R$. We make the further assumption that $b \ll p^2\,R^2$; this condition is satisfied in most cases of interest.

When $r' = R$, Eq. (11a) reduces to:

$$Z = a\ln\frac{R}{r} \pm [\omega^2 - p^2\,(R^2 - r^2)]^{\frac{1}{2}}\,, \tag{12}$$

where Z has been written for $\dot z_0$. This will be real if

$$r > \bar r = R\left[1 - \frac{\omega^2}{p^2\,R^2}\right]^{\frac{1}{2}}. \tag{13}$$

All the values of Z given by Eq. (12) are not allowed, since we must have (see Thomson [4]):

$$|Z| \leq \omega\,. \tag{14}$$

4. Total number of deuterons reaching the wall

When $b\log(r'/r)$ can be neglected in comparison with $p^2\,(r'^2 - r^2)$, Eq. (11a) can be written

$$Z = a\ln\frac{r'}{r} \pm [\omega^2 - p^2\,(r'^2 - r^2)]^{\frac{1}{2}}\,. \tag{15}$$

It will be seen that for $r' > R$ the two values of Z lie between the values Z_1, Z_2 of Z given by Eq. (12). If all the allowed values of Z are equally probable, the chance that a particle hits the wall is

$$\frac{Z_1 - Z_2}{2\omega}\,, \tag{16}$$

provided again $|Z_1| < \omega$ and $|Z_2| < \omega$. This can be shown to be true for most cases of interest, in particular if H_1 is large enough.

Therefore the chance that the deuteron hits the wall is

$$\frac{2\,[\omega^2 - p^2\,(R^2 - r^2)]^{\frac{1}{2}}}{2\omega} = \left[1 - \frac{p^2}{\omega^2}\,(R^2 - r^2)\right]^{\frac{1}{2}}$$
$$= \frac{p}{\omega}\,(r^2 - \bar r^2)^{\frac{1}{2}}. \tag{17}$$

If n is the density of deuterons at any point and $\overline{\sigma_s w}$ the average value of the product of scattering cross-section and relative velocity of deuterons, the total number of collisions at the point is

$$\frac{1}{2}\,n^2\,\bar\sigma_s w\,. \tag{18}$$

Thus the total number of deuterons reaching the wall per second will be

$$N = \int_{\bar r}^{R} n^2\,\overline{\sigma_s w}\,\frac{p}{\omega}\,\sqrt{r^2 - \bar r^2}\;2\pi\,r\,\mathrm{d}r\,. \tag{19}$$

We assume that n is given by the Boltzmann law

$$n = n_0\exp\left[-\frac{2\,\sigma\,e}{\mathrm{k}\,T}\,\ln\frac{r}{r_0}\right] = n_0\left(\frac{r}{r_0}\right)^{-\alpha}, \tag{20}$$

where

$$\alpha = 2\,\sigma\,e/\mathrm{k}\,T\,.$$

We can also write Eq. (20) as

$$n = n_R\left(\frac{r}{R}\right)^{-\alpha}. \tag{20a}$$

Putting $\varkappa = r/R$ and $\bar\varkappa = \bar r/R$, Eq. (19) becomes

$$N = 2\pi\,\overline{\sigma_s w}\,n_R{}^2\,\frac{p\,R^3}{\omega}\int_{\bar\varkappa}^{1}\varkappa^{-2\alpha+1}\sqrt{\varkappa^2 - \bar\varkappa^2}\,\mathrm{d}\varkappa\,. \tag{21}$$

If $\omega/p\,R = 2\,m\,\omega/e\,H_1$ is small compared with unity, $\bar{r}$ will be close to R and $\bar{\varkappa}$ will be close to unity. In this case the integral occurring in Eq. (21) will be of the order $\beta\,(\omega/p\,R)^3$, with a coefficient $\beta = 1/3$, if $\alpha = 0$ (i.e. in the absence of a radial electrostatic field); the coefficient will be somewhat larger than 1/3 if $\alpha > 0$. (Values of α can be estimated to be of the order of unity.)

Thus one obtains

$$N = 2\pi\beta\,\overline{\sigma_S\,w}\,n_R^2\,R^2\left(\frac{\omega}{p\,R}\right)^2. \qquad (22)$$

5. Comparison of the heat loss to that calculated by Thomson

We would like to compare our results to those obtained by G. P. Thomson [4]. For rather weak axial currents the containment depends upon the electrostatic field and Thomson finds ([4], see cases 1, 2 on page 891):

$$(N_T)_1 = 2\pi\,\overline{\sigma_S\,w}\,n_R^2\,R^2/\alpha = 2\pi\,\overline{\sigma_S\,w}\,n_R^2\,R^2\,(\mathrm{k}\,T/2\,\sigma e). \quad (23)$$

For strong axial currents, the electrostatic field is not important and could even be zero; for this case Thomson finds (case 3 and Section 9)

$$(N_T)_3 = 2\pi\,\overline{\sigma_S\,w}\,n_R^2\,R^2\,\omega/a = 2\pi\,\overline{\sigma_S\,w}\,n^2_R\,R^2\,(m\,\omega/2\,I e)\,. \quad (24)$$

When a strong axial magnetic field is present our result, Eq. (22), gives

$$N_J = 2\pi\,\beta\,\overline{\sigma_S\,w}\,n_R^2\,R^2\,(\omega/pR)^2 = 2\pi\beta\,\overline{\sigma_S\,w}\,n_R^2\,R^2\,(2m\,\omega/eH_1\,R)^2\,. \quad (22)$$

We find

$$\frac{N_J}{(N_T)_1} = \left(\frac{\omega}{pR}\right)^2\beta = \left(\frac{2\,m\,\omega}{e H_1\,R}\right)^2\frac{2\,\sigma e}{\mathrm{k}\,T}\,\beta\,. \qquad (23)$$

For Zeta, Thomson gives the data ([4] p. 894)

$I \sim 2 \times 10^4$ e.m.u (2×10^5 amps),

$T \sim 5 \cdot 10^6$ degrees K.

Thomson's figure $\omega \sim 6 \times 10^6$ does not quite correspond to this temperature: from $\frac{1}{2}\,m\,\omega^2 = \mathrm{k}\,T$ (as given by Thomson, p. 889) we find $\omega \sim 2 \times 10^7$ cm/sec. Thomson further mentions $\sigma = 10^{10}$—10^{11} (we shall take $\sigma = 3 \times 10^{10}$); $\pi\,r_0^2 \sim 100$ cm^2, from which $r_0 \sim 5.6$ cm; $\ln\,(R/r_0) \sim 1$, hence we assume $R = 20$ cm. We take $H_1 = 10^4$ gauss. With $m = 3.3 \times 10^{-24}$ gm for a deuteron and $e = 1.6 \times 10^{-20}$ e.m.u. we find $\alpha \sim 1.4$, $p = 2.4 \times 10^7$, $(\omega/p\,R) \simeq 4.2 \times 10^{-2}$ and $N_J/(N_T)_1 \sim \sim 2.4 \times 10^{-3}$.

Similarly

$$\frac{N_J}{(N_T)_3} = \left(\frac{\omega}{pR}\right)^2\frac{a}{\omega}\,\beta = \frac{8\,I\,m\,\omega}{e\,H_1^2\,R^2}\,\beta\,. \qquad (24)$$

With the data given above:

$$a \simeq 1.9 \times 10^8\,,$$
$$\frac{N_J}{(N_T)_3} \simeq 1.7 \times 10^{-2}\,. \qquad (25)$$

This shows that an axial magnetic field of 10^4 gauss considerably reduces the loss of particles to the wall. We note that, with the data used here,

$$b \simeq 5.8 \times 10^{14}\,,$$
$$p^2\,R^2 \simeq 2.3 \times 10^{17}\,, \qquad (26)$$

so that the assumption $b \ll p^2\,R^2$, made in order to simplify Eq. (11a), is justified.

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NUCLEAR FUSION 1 (1961) 201–202

LETTER TO EDITOR

THERMONUCLEAR REACTION RATES*

Physicists working on controlled thermonuclear reactions in the laboratory have had little occasion to consult tables of $\langle \sigma v \rangle$ so far. Happily, the indications are that this situation will soon be changed. To provide some measure of consistency, newly revised curves of $\langle \sigma v \rangle$ for DD, DT and DHe^3 are offered, see Figure 1 on next page. The results given here supersede the writer's ealier compilation [1], itself a revision of a still earlier one.

The $\langle \sigma v \rangle$ values are based on cross-sections which comprise in part the values given in Table I. The values of σ below 120 keV are those of ARNOLD, PHILLIPS, SAWYER, STOVALL and TUCK [2] except that the neutron branch DD_n has been modified to take into account the angular distribution of PRESTON, SHAW and YOUNG [3] in accordance with the method of MCNEILL [4]. The latter moves $\sigma(DD_n)/\sigma(DD_p)$ upward at low energies so that the branching ratio now declines to unity at the lowest energies, instead of to 0.92. The values of σ at energies above 120 keV come from many sources tabulated elsewhere [5].

The Maxwell averages given here are in close agreement with values given at 5, 10, 25, 50, 100 and 150 keV temperatures by HESSELBERG JENSEN, KOFOED-HANSEN, SILLESEN and WANDEL [6].

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TABLE I Basic cross-section values

Deuteron energy in laboratory system (keV)	σ in millibarns			
	DD_n	DD_p	DT	DHe^3
15	0.065	0.065	15	
30	1.15	1.10	278	
60	6.8	6.5	2180	2.3
120	23	20	4700	31
250	46	38	1720	290
500	74	59	660	630
1000	96	78	280	230

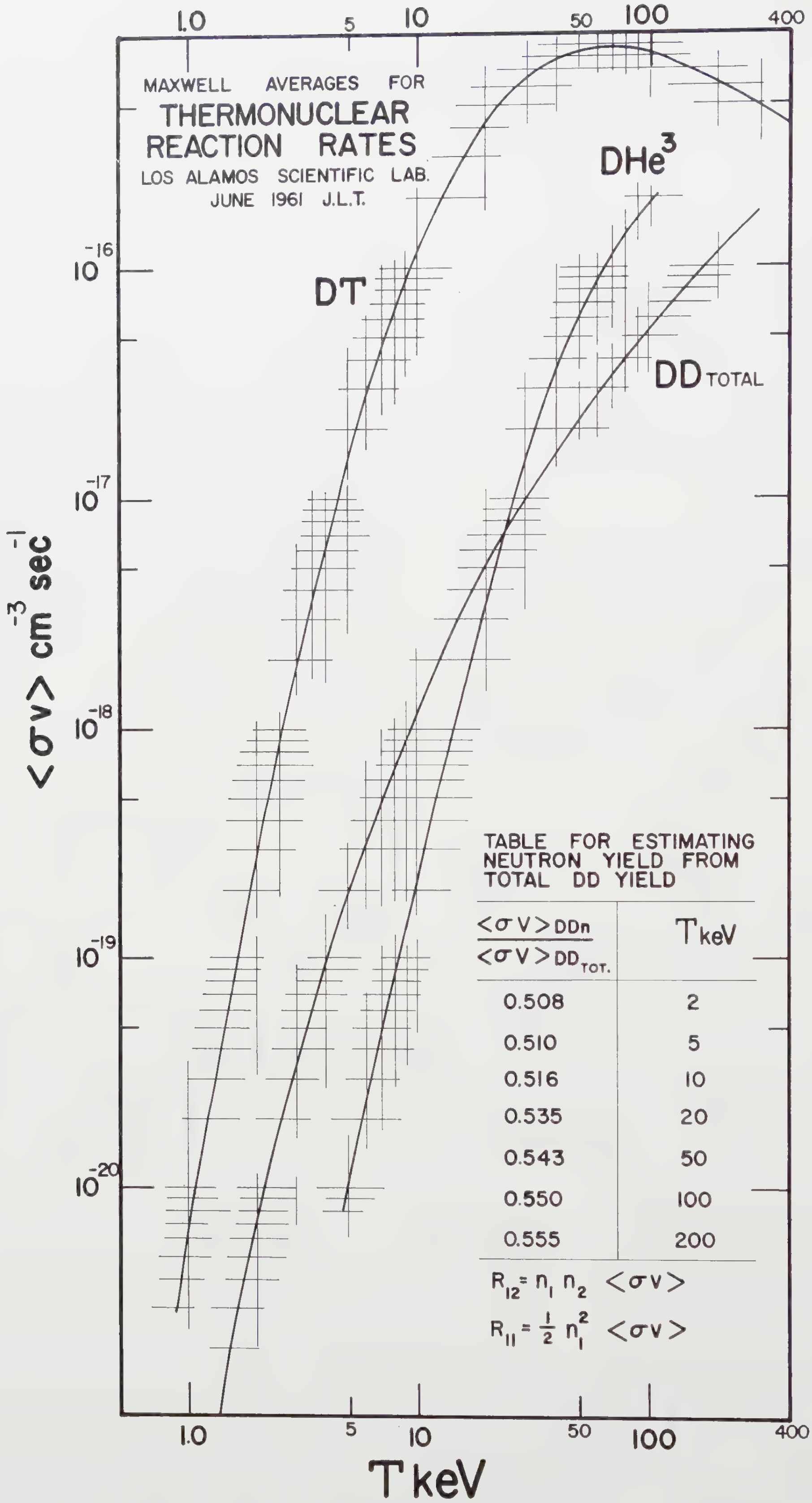

Fig. 1. Maxwell averages for DD_{TOTAL}, DT and DHe^3 reaction rates as a function of incident deuteron energy in keV.

Behaviour of plasma in a rotating magnetic field, A. LEGATOWICZ *(Institute of Nuclear Research, Warsaw, Poland)*

Nuclear Fusion 1 (1961) 155—159

An investigation is made of the non-relativistic motion of a charged particle (in a plasma) in an external rotating electromagnetic field of the form: $H_x = H_0 \cos(\omega y/c) \cos \omega t$, $H_y = H_0 \cos(\omega x/c) \sin \omega t$, $H_z = \text{constant}$, $E_x = E_y = 0$, $E_z = H_0 [\sin(\omega x/c) \cos \omega t + \sin(\omega y/c) \sin \omega t]$, with $\omega x/c \ll 1$, $\omega y/c \ll 1$. If the condition $-1 < (eH_z/mc\omega) < [-1 + (eH_0/mc\omega)^2]$ is satisfied, then the particle moves away from the z-axis. The particle energy is $\sim H_0^2 e^2 \bar{r}^2/(mc^2 \text{k})$ in °K where $\bar{r}$ is the average distance from the z-axis.

The motion of the plasma is then investigated taking into account its proper electromagnetic field. The following transport equation is used: $nm\,\dot{\mathbf{v}} + \mathbf{v}\nabla \cdot \mathbf{v} = nq\,(\mathbf{E} + \mathbf{v} \times \mathbf{H}/c) - \nabla\psi - nm\,\nabla\varphi + \mathbf{p}$. The assumptions are: a plasma consisting of equal populations of electrons and deuterons with density $\sim 10^{15}/\text{cm}^3$, $\omega \simeq 10^{10}/\text{sec}$, $H_0 \sim 10^3$ G, $v = 0.1\,c$. At $t = 0$ it is assumed that $T = 10^6$ K and $v = 0$ and that the derivatives of the plasma density with respect to space variables are negligible compared to other terms of the transport equation. An expansion in powers of v/c is used. Zeroth, first and second order approximations are calculated using the Laplace transformation. Up to the second order of approximation, the field causes neither a durable change in plasma density nor a charge separation.

Oscillations of four different frequencies appear in the plasma. At a definite frequency of the rotating field there appears a resonance phenomenon in which the amplitude of oscillation increases linearly with t. At resonance the mean energy per ion (in °K) transferred directly to the ionic part of the plasma increases with time as follows: $(1/192\,\pi^2)\,(e^2 m/c^4 M^4\,\text{k})\,(H_0^4 H_z^2/n_0^2)\,t^2$. This means, for example, that in an axial field of 10^4 G and a rotating field of amplitude 10^3 G, the time necessary to provide energy corresponding to 10^8 °K (disregarding losses) is ~ 0.3 sec.

Axial conduction and radiation losses in a stabilised linear pinch, A. H. DE BORDE, F. A. HAAS *(English Electric Co. Ltd., Nelson Research Laboratories, Stafford, England)*

Nuclear Fusion 1 (1961) 160—166

This paper presents a theory of conduction and radiation losses in a linear pinched discharge under steady conditions. The model selected is one in which the ohmic heating in a thin skin of current is equated to the radiation and axial conduction losses, the discharge being considered at a uniform pressure determined by the Bennett relation modified to include the effect of a possible completely trapped axial magnetic field. Formulae for temperature and other physical quantities are presented and limiting forms considered in a variety of circumstances. The effect of thermoelectric phenomena is considered and conditions under which the treatment is likely to be applicable discussed.

Longitudinal oscillations in a neutralized electron beam with a boundary, M. YOSHIKAWA *(Department of Physics, University of Tokyo, Tokyo, Japan)*

Nuclear Fusion 1 (1961) 167—171

Longitudinal oscillation of a neutralized electron beam in a strong axial magnetic field is discussed. The beam is assumed to be cylindrical and to be confined in a conducting cylinder, or to be planar and to be held between two conducting plates. A sufficient and, in some cases, approximately necessary condition for stability is obtained. To be stable, the beam current should be below a certain value which depends on the electron velocity, the ratio of electron to ion mass and the geometrical dimensions of the beam and the conductor. The validity of the approximation is also studied.

Radiation from a modulated beam of charged particles penetrating a plasma in a uniform magnetic field, E. CANOBBIO *(Max-Planck-Institut für Physik und Astrophysik, Munich, Federal Republic of Germany)*

Nuclear Fusion 1 (1961) 172—180

The radiation from a density-modulated beam of ions, which penetrates a plasma perpendicular to a strong magnetic field $\mathbf{B}_0$, is investigated in two simplified situations: *a*) the beam is an infinite plane parallel to $\mathbf{B}_0$, and *b*) the beam is an infinite cylindrical surface parallel to $\mathbf{B}_0$, the radius of the cylinder being the gyro-radius of the beam particles. This latter beam can be ideally constructed by injecting into a plasma a linear beam, modulated at a frequency which is an integral multiple of the gyrofrequency of the beam particles and incident in a direction which forms a very small angle with a plane perpendicular to $\mathbf{B}_0$.

In both situations some resonances of the Poynting vector are found. The resonance, which occurs when the modulation frequency is equal to the "ion-resonance" frequency, is specifically investigated, taking into account the finite electric conductivity of the plasma. It is shown that, under appropriate conditions, the beam-plasma interaction at this resonance becomes very strong.

Theory of Čerenkov and cyclotron radiations in plasmas, T. Kihara. O, Aono, R. Sugihara *(Department of Physics, University of Tokyo, Tokyo, Japan)* Nuclear Fusion 1 (1961) 181—188

Radiation from a charge q moving in a helix in magneto-plasma is investigated theoretically. When its speed v is much larger than the thermal velocity $(m^{-1}\,\mathrm{k}T)^{1/2}$ of the plasma electrons and the gyration frequency is much smaller than the plasma frequency ω_0, the radiation power from the charge is $(q^2\,\omega^2{}_0/2v)\,\ln\,(v^2/m^{-1}\,\mathrm{k}T)$. Cyclotron radiation from an electron with non-relativistic speed decreases to zero as plasma density increases. For a positron in a dilute plasma, however, the radiation is strengthened. This strengthened radiation from a positron again decreases with increasing $\omega^2{}_0/\omega_\mathrm{H}{}^2$ and becomes zero for $\omega^2{}_0/\omega_\mathrm{H}{}^2 \geq 2$ (ω_H = gyration frequency of the plasma electron). Damping of the Čerenkov radiation due to collisions of plasma electrons is also discussed.

Cyclotron radiation by ions in plasma, V. I. Pistunovich, V. D. Shafranov *("I. V. Kurchatov" Institute of Atomic Energy, USSR Academy of Sciences, Moscow, USSR)* Nuclear Fusion 1 (1961) 189—194

In this work the intensity of radiation by fast ions in cold plasma is determined. This problem arose in connection with observations made on the OGRA device of resonance peaks of the electric field strength, i.e. at the ion cyclotron frequency and its overtones. Although the abovementioned observations were made outside the wave zone, it is natural to expect that the experimentally revealed dependence of the number of observed peaks on the density of cold plasma must also be reflected in the intensity of radiation. Calculations showed that, with an increase of the ion velocity and of the plasma density, the maximum of the intensity of radiation actually shifts towards the high frequencies.

This shift is analogous to the shift of the maximum of the radiation intensity for synchrotron radiation by an electron which has a velocity approaching the velocity of light. In the case of ions in plasma the role of the velocity of light is played by the phase velocity of the electro-magnetic waves in the plasma. This velocity approaches the Alfvén velocity $c_\mathrm{A} = B_0/\sqrt{4\,\pi\,m_\mathrm{i}\,n_0}$ in the region which is important in the calculations. Therefore, the high overtones already become effective at a comparatively small ion velocity, $0 \sim c_\mathrm{A} \ll c$.

Measurement of the plasma electron temperature in a strong shock wave, T. I. Filippova, N. V. Filippov, V. V. Zhurin, V. P. Vinogradov *("I. V. Kurchatov" Institute of Atomic Energy, USSR Academy of Sciences, Moscow, USSR)* Nuclear Fusion 1 (1961) 195—197

The electron conductivity of deuterium plasma behind a strong shock wave has been measured by the method of displaced magnetic flux. At shock-wave velocities from 0.9×10^7 to 1.25×10^7 cm/sec electron temperatures of 50 and 90 eV have been found. A comparison between the observed electron temperature and the electron temperature computed from the shock-wave velocity has been made. Piezoelectric measurements and compression of plasma by a strong magnetic field yield exactly the same value of the gas-kinetic pressure of plasma.

Loss of particles in a pinched discharge in an axial magnetic field, R. K. Jaggi *(Institute for Fluid Dynamics and Applied Mathematics University of Maryland, College Park, Maryland, U.S.A.)*

Nuclear Fusion 1 (1961) 198—200

A calculation of the loss of deuterons from a pinched current to the wall of the container has been given by G. P. Thomson [Phil. Mag. **32** (1958) 886]. This calculation is extended so that account is taken of an axial magnetic field. It is found that such a field can materially reduce the particle loss.

RÉSUMÉS EN FRANÇAIS

Comportement du plasma dans un champ magnétique tournant, A. Legatowicz *(Institut de recherche nucléaire, Varsovie, Pologne)*
Fusion Nucléaire 1 (1961) 155—159

L'auteur étudie le mouvement non relativiste d'une particule chargée (dans un plasma) dans un champ électromagnétique tournant externe de la forme: $H_x = H_0 \cos(\omega y/c) \cos \omega t$, $\mathrm{H}_y = H_0 \cos(\omega x/c) \sin \omega t$, $H_z =$ constant, $E_x = E_y = 0$, $E_z = H_0 [\sin(\omega x/c) \cos \omega t + \sin(\omega y/c) \sin \omega t]$, avec $\omega x/c \ll 1$, $\omega y/c \ll 1$. Si la condition $-1 < (eH_z/mc\omega) < [-1 + (eH_0/mc\omega)^2]$ est réalisée, la particule s'éloigne de l'axe z. L'énergie de la particule dans °K est $\sim H_0\, e^2 \bar{r}/mc^2$ k), où $\bar{r}$ est la distance moyenne à partir de l'axe z.

L'auteur étudie ensuite le mouvement du plasma en tenant compte de son champ électromagnétique propre. A cet régard, il utilise l'équation de transport: $nm\ \dot{\mathbf{v}} + \mathbf{v}\nabla \cdot \mathbf{v} = \mathrm{nq}\ (\mathbf{E} + \mathbf{v} \times \mathbf{H}/c) - \nabla\psi - nm\ \nabla\varphi + \mathbf{p}$. Il admet les hypothèses suivantes: le plasma est composé de populations égales d'électrons et de deutérons d'une densité de $\sim 10^{15}/\mathrm{cm}^3$, $\omega \lesssim 10^{10}/\mathrm{s}$, $H_0 \sim 10^3$ G, $v \leq 0{,}1\ c$; pour $t = :0$, $T = 10^6$ °K et $v = 0$ et les dérivées de la densité du plasma par rapport aux variables spatiales sont négligeables si on les compare aux autres termes de l'équation de transport. Il utilise un développement en série de puissances de v/c. Il calcule les approximations d'ordre zéro, un et deux au moyen de la transformation de Laplace. Jusqu'à l'approximation du deuxième degré, le champ ne donne lieu ni à un changement durable dans la densité du plasma ni à une séparation de charges.

Des oscillations de quatre fréquences différentes apparaissent dans le plasma. A une fréquence déterminée du champ tournant, on constate un phénomène de résonance dans lequel l'amplitude de l'oscillation augmente d'une façon linéaire avec le temps. A la résonance, l'énergie moyenne par ion (en °K) transférée directement à la partie ionique du plasma augmente avec le temps comme suit: $(1/192\ \pi^2)\ e^2\ m/c^4\ M^4$ k) $H_0^4\ H_z^2/n_0^2)\ t^2$. Cela signifie, par exemple, que, dans un champ axial de 10^4 G et un champ tournant d'une amplitude de 10^3 G, le temps nécessaire pour fournir une énergie correspondant à 10^8 °K (abstraction faite des pertes) est $\sim 0{,}3$ secondes.

Pertes dues à la conduction axiale et au rayonnement dans une striction linéaire stabilisée, A. H. de Borde et F. A. Haas *(English Electric Co. Ltd., Nelson Research Laboratories, Stafford, Royaume-Uni)*
Fusion Nucléaire 1 (1961) 160—166

Le mémoire expose une théorie concernant les pertes dues à la conducton et au rayonnement dans une décharge linéaire pincée, dans des conditions stables. Le modèle choisi est conçu de telle sorte que le chauffage ohmique dans une mince couche de courant compense les pertes dues au rayonnement et à la conduction axiale, la décharge étant considérée à une pression uniforme qui est déterminée par le rapport Bennett modifié de manière à tenir compte de l'effet d'un champ magnétique axial éventuel entièrement capturé. Les auteurs présentent des formules concernant la température et d'autres quantités physiques et examinent les facteurs de limitation dans diverses circonstances. Les auteurs examinent enfin l'effet des phénomènes thermoélectriques et les conditions dans lesquelles le traitement est susceptible d'application.

Oscillations longitudinales dans un faisceau d'électrons neutralisé et limité, M. Yoshikawa *(Département de physique, Université de Tokyo, Tokyo, Japon)*
Fusion Nucléaire 1 (1961) 167—171

L'auteur examine les oscillations longitudinales d'un faisceau d'électrons neutralisé dans un champ magnétique axial intense. Il suppose le faisceau cylindrique et contenu dans une surface cylindrique conductrice, ou de forme plane et compris entre deux plaques conductrices. Il réussit à déterminer pour la stabilité une condition suffisante et, dans certains cas, presque nécessaire. Pour que la stabilité soit réalisée, le courant du faisceau doit être inférieur à une certaine valeur qui dépend de la vitesse des électrons, du rapport entre la masse des électrons et celle des ions, ainsi que des dimensions géométriques du faisceau et du conducteur. L'auteur étudie également la validité de l'approximation.

Rayonnements émis par un faisceau modulé de particules chargées pénétrant dans un plasma dans un champ magnétique uniforme, E. Canobbio *(Max-Planck-Institut für Physik und Astrophysik, République fédérale d'Allemagne)*
Fusion Nucléaire 1 (1961) 172—180

Les rayonnements émis par un faisceau d'ions à densité modulée, qui pénètre dans un plasma perpendiculaire à un champ magnétique intense $\mathbf{B}_0$, sont étudiés dans deux situations simplifiées: *a)* le faisceau est un plan infini parallèle à $\mathbf{B}_0$; *b)* le faisceau est une surface cylindrique infinie, parallèle à $\mathbf{B}_0$, dont le rayon est le rayon de rotation des particules du faisceau. On peut construire idéalement ce dernier faisceau en injectant dans un plasma un faisceau linéaire, modulé à une fréquence qui est un multiple entier de la fréquence de giration des particules du faisceau, et tombant suivant une direction qui forme un très petit angle avec un plan perpendiculaire à $\mathbf{B}_0$.

Dans les deux situations, on observe quelques résonances du vecteur de Poynting. L'auteur étudie plus particulièrement la résonance qui se produit lorsque la fréquence de modulation est égale à la fréquence de la «résonance ionique», en tenant compte de la conductibilité électrique finie du plasma. Il démontre que, dans des conditions appropriées, l'interaction faisceau-plasma devient très forte à cette résonance.

La théorie du rayonnement de Tcherenkov et du rayonnement cyclotronique dans un plasma, T. Kihara, O. Aono et R. Sugihara *(Départment de physique, Université de Tokyo, Tokyo, Japon)*
Fusion Nucléaire **1** (1961) 181—188

Les auteurs étudient, du point de vue théorique, les rayonnements émis par une charge q se déplaçant en hélice dans des magnétoplasmas. Lorsque la vitesse v est de beaucoup supérieure à la vitesse thermique $(m^{-1}\,kT)^{1/2}$ des électrons du plasma et que la fréquence de rotation est de beaucoup inférieure à la fréquence du plasma ω_0, l'énergie du rayonnement émanant de la charge est $(q^2\,\omega_0{}^2/2\,v)\ \ln\,(v^2/m^{-1}\,kT)$. L'intensité du rayonnement cyclotronique émis par un électron ayant une vitesse non relativiste tend vers zéro lorsque la densité du plasma augmente. Cependant, dans le cas d'un positon se trouvant dans un plasma dilué, le rayonnement se trouve intensifié. Ce rayonnement plus intense émis par un positon diminue ensuite à mesure que $\omega^2{}_0/\omega_H{}^2$ augmente et tombe à zéro pour $\omega^2{}_0/\omega_H{}^2 \geq 2$ ($\omega_H =$ la fréquence de rotation de l'électron plasmatique). Les auteurs examinent en outre l'amortissement du rayonnement de Tcherenkov par suite de collisions d'électrons plasmatiques.

Rayonnement cyclotronique des ions dans un plasma, V. I. Pistounovitch et V. D. Shafranov *(Institut I. V. Kourtchatov de l'énergie nucléaire, Académie des sciences de l'Union soviétique, Moscou, URSS)*
Fusion Nucléaire **1** (1961) 189—194

Dans cette étude les auteurs déterminent l'intensité du rayonnement des ions rapides dans un plasma froid. Le problème s'est posé à propos de l'observation, au moyen d'un dispositif OGRA à des régimes déterminés, des pics de résonance de l'intensité du champ électrique à partir de la fréquence cyclotronique des ions et de ses harmoniques supérieurs. Bien que les observations mentionnées n'aient pas été faites dans la zone de l'onde, il était naturel de prévoir que la relation de dépendance, révélée par l'expérience, entre le nombre de pics observés et la densité du plasma froid se refléterait aussi dans l'intensité du rayonnement. Les calculs ont montré que lorsque la vitesse des ions et la densité du plasma augmentent, l'intensité maximum se déplace effectivement vers les hautes fréquences.

Ce déplacement est analogue à celui du maximum d'intensité du rayonnement synchrotronique dû à un électron ayant une vitesse proche de celle de la lumière. Lorsqu'il s'agit d'ions dans un plasma, la rôle de la vitesse de la lumière est joué par la vitesse de phase des ondes électromagnétiques dans le plasma; dans la région importante pour les calculs, cette vitesse s'approche de la vitesse d'Alfvén: $c_A = B_0/(4\,\pi\,m_i\,n_0)^{1/2}$.
Il s'ensuit que les harmoniques supérieurs élevés se manifestent déjà à une vitesse relativement faible des ions: $v \sim c_A \ll c$.

Mesure de la température électronique du plasma dans une forte onde de choc, T. I. Filippova, N. V. Filippov, V. V. Jourin et V. P. Vinogradov *(Institut I.V. Kourtchatov de l'énergie nucléaire, Académie des sciences de l'Union soviétique, Moscou, URSS)*
Fusion Nucléaire **1** (1961) 195—197

La conductivité électronique du plasma au deutérium, à l'arrière d'une forte onde de choc, a été mesurée par la méthode du flux magnétique déplacé. On a constaté qu'aux vitesses de propagation de l'onde de choc comprises entre $0{,}9 \times 10^7$ et $1{,}25 \times 10^7$ cm/s, correspondaient des températures électroniques de 50 à 90 eV. La température électronique observée a été comparée à celle que l'on a calculée à partir de la vitesse de l'onde de choc. Les mesures piézo-électriques et la compression du plasma par un champ magnétique intense ont donné une seule et même valeur pour la pression cinétique des gaz du plasma.

Perte de particules au cours d'une décharge pincée dans un champ magnétique axial, R. K. Jaggi *(Institut de la dynamique des fluides et de mathématiques appliquées, Université du Maryland, College Park, Maryland, Etats-Unis)*
Fusion Nucléaire **1** (1961) 198—200

La perte de deutérons entre le courant pincé et la paroi de la chambre a été calculée par G. P. Thomson [Phil. Mag. **32** (1958) 886]. L'auteur du présent article a perfectionné le calcul de manière à tenir compte de la présence d'un champ magnétique axial. On a constaté qu'un tel champ peut réduire sensiblement la perte de particules.

АННОТАЦИЯ ПО-РУССКИ

Поведение плазмы во вращающемся магнитном поле, А. ЛЕГАТОВИЧ *(Институт ядерных исследований, Варшава, Польша)*

Ядерный синтез **1** (1961) 155—159

Проведено исследование нерелятивистского движения заряженной частицы (в плазме) во внешнем вращающемся электромагнитном поле следующего вида: $H_x = H_0 \cos(\omega y/c) \cos \omega t$, $H_y = H_0 \cos(\omega x/c) \sin \omega t$, $H_z = \text{const.}$, $E_x = E_y = 0$, $E_z = H_0 \, [\sin(\omega x/c) \cos \omega t + \sin(\omega y/c) \sin \omega t]$, $\omega x/c \ll 1$, $\omega y/c \ll 1$. Если удовлетворяется условие $-1 < (e\,Hx/mc\omega) < [-1 + (e\,H_0/mc\omega)^2]$, то частица передвигается от оси-z. Энергия частицы в градусах шкалы Кельвина составляет $\sim H_0\, e^2 \bar{r}^{\,2}/(m\,c^2\,\mathrm{k})$, где $\bar{r}$ среднее расстояние от оси-z.

Затем было исследовано движение плазмы с учетом ее соответствующего электромагнитного поля. Применялось следующее уравнение переноса:
$n\,m\,\dot{\mathbf{v}} + \mathbf{v}\nabla\cdot\mathbf{v} = n\,q\,(\mathbf{E} + \mathbf{v}\times\mathbf{H}/c) - \nabla\psi - n\,m\,\nabla\varphi + \mathbf{p}$. Предположениями являются: плазма, состоящая из равного числа электронов и дейтронов с плотностью $\sim 10^{15}/\text{см}^{-3}$, $\omega \lesssim 10^{10}$ сек, $H_0 \sim 10^3$ гаусс $v \leq 0{,}1\ c$. При $t = 0$ предполагается, что $T = 10^6$ °K и $v = 0$ и что производными плотности плазмы по пространственным переменным можно пренебречь по сравнению с другими членами уравнения переноса. Используется разложение в ряд по степеням v/c. С помощью преобразования Лапласа подсчитываются приближенные величины нулевого, первого и второго порядка. Вплоть до второго порядка приближения поле не вызывает ни длительного изменения в плотности плазмы, ни разделения заряда.

В плазме появляются осцилляции четырех различных частот. При определенной частоте вращающегося поля появляется явление резонанса, при котором амплитуда осцилляции возрастает линейно со временем. При явлении резонанса средняя энергия на ион (в градусах шкалы Кельвина), передаваемая непосредственно в ионную часть плазмы, возрастает со временем следующим образом: $(1/192\,\pi^2)\,(e^2 m/c^4 M^4\,\mathrm{k})\,(H_0^4\,H_z^2/n_0^2)\,t^2$. Это означает, например, что в аксиальном поле 10^4 гаусс и во вращающемся поле амплитуды 10^3 гаусс, время необходимое для достижения энергии порядка 10^8 в градусах шкалы Кельвина (без учета потерь), составляет $\sim 0{,}3$ сек.

Аксиальная проводимость и радиационные потери в стабилизированном линейном пинч-эффекте, А. Х. ДЕ БОРД Ф. А. ХААС *(Инглиш Электрик К°, Исследовательские лаборатории имени Нельсона, Стаффорд, Англия)*

Ядерный синтез **1** (1961) 160—166

В настоящем докладе изложена теория проводимости и радиационных потерь в линейном сжатом разряде при устойчивых условиях. В избранной модели омическое нагревание в тонком слое тока приравнивается к аксиальной проводимости и радиационным потерям, причем разряд рассматривается при однородном давлении, определяемом соотношением Беннетта, измененным с целью включения эффекта как можно более вмороженного в плазму аксиального магнитного поля. Представлены формулы для температуры и для других физических величин; рассматриваются ограничивающие формы в различных условиях. Рассматривается эффект термоэлектрических явлений и обсуждаются условия, при которых, по-видимому, можно применить данный метод.

Продольные колебания в нейтрализованном электронном пучке с границей, М. ИОСИКАВА *(Физический отдел, Токийский университет, Токио, Япония)*

Ядерный синтез **1** (1961) 167—171

Обсуждается продольное колебание нейтрализованного электронного пучка в мощном аксиальном магнитном поле. Пучок предполагается цилиндрическим и ограниченным в проводящей цилиндрической оболочке или плоским и удерживаемым между двумя проводящими пластинками. Создается достаточное, а в некоторых случаях почти необходимое условие для устойчивости. Чтобы добиться устойчивости, ток пучка должен быть ниже определенной величины, которая зависит от скорости электронов, соотношения электронной и ионной масс, а также от геометрических размеров пучка и проводника. В докладе изучается также обоснованность приближения.

Излучение из модулированного пучка заряженных частиц, проникающего в плазму в однородном магнитном поле, Е. КАНОББИО *(Физический и астрофизический институт Макса-Планка, Мюнхен, ФРГ)*

Ядерный синтез **1** (1961) 172—180

Исследуется излучение от пучка ионов модулированной плотности, который проникает через плазму перпендикулярно сильному магнитному полю B_0 в двух упрощенных положениях: а) пучок является бесконечной плоскостью, параллельной B_0; и б) пучок является бесконечной цилиндрической поверхностью, параллельной B_0, радиус которой является радиусом вращения частиц пучка. Этот последний пучок может быть идеально создан введением в плазму линейного пучка, модулированного при частоте, которая является целым кратным частоты вращения частиц пучка, и падающего в направлении, которое создает очень малый угол с плоскостью, перпендикулярной B_0.

В обоих положениях найдено несколько резонансов вектора Пойнтинга. Специально исследуется резонанс, получаемый в том случае, когда модуляционная частота равна «ион-резонансной» частоте, с учетом ограниченной электропроводимости плазмы. Найдено, что в соответствующих условиях взаимодействие между пучком и плазмой при этом резонансе становится очень сильным.

Теория черенковского и циклотронного излучения в плазме, Т. КИХАРА, О. АОНО, Р. СУГИХАРА *(Физический отдел, Токийский университет, Токио, Япония)* Ядерный синтез **1** (1961) 181—188

Теоретически исследуется излучение заряда q, движущегося по спирали в магнитоплазмах. Когда его скорость v значительно превышает тепловую скорость $(m^{-1}kT)^{1/2}$ электронов плазмы, а частота вращения значительно меньше частоты плазмы ω_0, мощность радиации заряда равна $(q^2\omega_0^2/2v)\ \ln\ (v^2/m^{-1}\ kT)$. Циклотронная радиация электрона двигающегося с нерелятивистской скоростью понижается до нуля с увеличением плотности плазмы. Однако для позитрона в разреженной плазме интенсивность излучения увеличивается. Эта увеличенная интенсивность излучения позитрона вновь понижается с увеличением $\omega^2{}_0/\omega_H{}^2$ и становится нулевой для $\omega^2{}_0/\omega_H{}^2 \geq 2$ (ω_H — частота вращения электрона плазмы). Рассматривается также затухание черенковского излучения вследствие столкновений плазменных электронов.

Циклотронное излучение ионов в плазме, В. И. ПИСТУНОВИЧ, В. Д. ШАФРАНОВ *(Институт атомной энергии имени И. В. Курчатова Академии Наук СССР, Москва, СССР)* Ядерный синтез **1** (1961) 189—194

В работе определена интенсивность излучения быстрых ионов в холодной плазме. Эта задача возникла в связи с наблюдением на установке Огра в определенных режимах резонансных пиков напряженности электрического поля на циклотронной ионной частоте и ее обертонах. Хотя указанные наблюдения производились не в волновой зоне, естественно ожидать, что обнаруженная в эксперименте зависимость числа наблюдаемых пиков от плотности холодной плазмы должна найти свое отражение и в интенсивности излучения. Расчеты показали, что при увеличении скорости ионов и плотности плазмы максимум интенсивности излучения, действительно, смещается в сторону высоких частот.

Это смещение аналогично смещению максимума интенсивности излучения при синхротронном излучении электрона, имеющего скорость, близкую к скорости света. В случае ионов в плазме роль скорости света играет фазовая скорость электромагнитных волн в плазме, которая в существенной для расчетов области близка к альфвеновской скорости $c_A = B_0/(4\ \pi\ m_i\ n_0)^{1/2}$. Поэтому высокие обертоны проявляются уже при сравнительно небольшой скорости ионов $v \sim c_A \ll c$.

Измерение электронной температуры плазмы в мощной ударной волне, Т. И. ФИЛИППОВА, Н. В. ФИЛИППОВ, В. В. ЖУРИН, В. П. ВИНОГРАДОВ *(Институт атомной энергии имени И. В. Курчатова Академии Наук СССР, Москва, СССР)* Ядерный синтез **1** (1961) 195—197

Методом вытесненного магнитного потока измерена электронная проводимость дейтериевой плазмы за сильной ударной волной. При скоростях ударной волны от $0{,}9 \cdot 10^7$ до $1{,}25 \cdot 10^7$ см/сек найдена электронная температура в 50 и 90 eV. Проведено сравнение полученной электронной температуры с рассчитанной по скорости ударной волны. Пьезоэлектрические измерения и сжатие плазмы сильным магнитным полем дали одно и то же значение газокинетического давления плазмы.

Потеря частиц в разряде типа пинч в осевом магнитном поле, Р. К. ДЖАГГИ *(Институт динамики жидкостей и прикладной математики Мерилэндского университета, Колледж Парк, Мерилэнд, США)* Ядерный синтез **1** (1961) 198—200

Г. П. Томсоном [Phil. Mag. **32** (1958) 886] были сделаны расчеты потери дейтонов из пинч-тока к стенке сосуда. Эти расчеты распространены для учета влияния осевого магнитного поля. Найдено, что такое поле может существенно уменьшить потерю частиц.

Comportamiento del plasma en un campo magnético giratorio, A. LEGATOWICZ *(Instituto de Investigaciones Nucleares, Varsovia, Polonia)*
Fusión Nuclear 1 (1961) 155—159

El autor estudia el movimiento no relativista de una partícula cargada (en un plasma) en un campo electromagnético giratorio externo, que responde a la forma: $H_x = H_0 \cos (\omega y/c) \cos \omega t$, $H_y = H_0 \cos (\omega x/c)$ sen ωt, $H_z =$ constante, $E_x = E_y = 0$, $E_z = H_0$ [sen $(\omega x/c) \cos \omega t +$ sen $(\omega y/c)$ sen ωt], siendo $\omega x/c \ll 1$, $\omega y/c \ll 1$. Si se cumple la condición $-1 < (eH_x/mc\,\omega) < [-1 + (eH_0/mc\,\omega)^2]$, la partícula se aleja del eje z. La energía de la partícula, expresada en °K, es $\sim H_0 e^2 \bar{r}^2/(mc^2 \mathrm{k})$, donde $\bar{r}$ es la distancia media al eje z.

Seguidamente, el autor estudia el movimiento del plasma teniendo en cuenta su campo electromagnético propio. Utiliza la siguiente ecuación de transporte: $nm\dot{\mathbf{v}} + \mathbf{v}\nabla\cdot\mathbf{v} = nq\,(\mathbf{E} + \mathbf{v} \times \mathbf{H}/c) - \nabla\psi - nm\,\nabla\varphi + \mathbf{p}$, formulando las siguientes hipótesis básicas: el plasma consta de número igual de electrones y deuterones con una densidad de $\sim 10^{15}/\mathrm{cm}^3$, $\omega \lesssim 10^{10}/\mathrm{s}$, $H_0 \sim 10^3$ G, $v \leq 0{,}1\,c$. Para $t = 0$, se supone que $T = 10^6$ °K y $v = 0$ y que las derivadas de la densidad del plasma con respecto a las variables espaciales son despreciables en comparación con los demás términos de la ecuación de transporte. Desarrolla v/c en serie de potencias y calcula las aproximaciones de orden cero, primero y segundo, usando la transformación de Laplace. Hasta la aproximación de segundo orden, el campo no causa cambios duraderos en la densidad del plasma ni provoca ninguna separación de cargas.

Se observan en el plasma oscilaciones de cuatro frecuencias diferentes. Cuando la frecuencia del campo giratorio alcanza un valor determinado, se manifiesta un fenómeno de resonancia en el que la amplitud de la oscilación crece en forma directamente proporcional al tiempo. Durante la resonancia, la energía media por ion (expresada en °K) que se transfiere directamente a la parte iónica del plasma aumenta con el tiempo según la siguiente función: $(1/192\,\pi^2)\,(e^2m/c^4\,M^4\,\mathrm{k})\,H^4{}_0\,H^2{}_z/n_0{}^2)\,t^2$. Esto significa que, por ejemplo, en un campo axial de 10^4 G y un campo giratorio de una amplitud 10^3 G el tiempo necesario para suministrar la energía correspondiente a 10^8 °K (despreciando las pérdidas) es del orden de los ~ 0.3 s.

Pérdidas debidas a la conducción axial y a la radiación en una estricción lineal estabilizada, A. H. DE BORDE, F. A. HAAS *(English Electric Co. Ltd., Nelson Research Laboratories, Stafford, Reino Unido)*
Fusión Nuclear 1 (1961) 160—166

Los autores exponen una teoría acerca de las pérdidas debidas a la conducción y a la radiación en una descarga con estricción lineal en condiciones de estabilidad. Han elegido un modelo en el que se admite que el calentamiento óhmico en una capa delgada de corriente compensa las pérdidas por radiación y por conducción axial, considerándose que la descarga se verifica a una presión uniforme que está determinada por la relación de Bennett, modificada a fin de tener en cuenta el efecto de un posible campo magnético axial totalmente capturado. Presentan fórmulas referentes a la temperatura y otros parámetros físicos y consideran factores de limitación en diversas circunstancias.
Por último, estudian el efecto de los fenómenos termoeléctricos y analizan las condiciones en que el tratamiento es susceptible de aplicarse.

Oscilaciones longitudinales en un haz electrónico neutralizado y limitado, M. YOSHIKAWA *(Departamento de Física, Universidad de Tokio, Tokio, Japón)*
Fusión Nuclear 1 (1961) 167—171
El autor estudia las oscilaciones longitudinales de un haz de electrones neutralizado en un campo magnético axial de gran intensidad. Supone que el haz es cilíndrico y se encuentra encerrado en un cilindro conductor o bien que es de forma plana y está intercalado entre dos placas conductoras. Determina la condición suficiente y, en algunos casos, cuasi necesaria, para que se mantenga la estabilidad. Para alcanzar esa estabilidad, es preciso que la corriente del haz sea inferior a cierto valor, que depende de la velocidad de los electrones y de la razón masa electrónica/masa iónica, así como de las dimensiones geométricas del haz y del conductor. El autor estudia también la validez de la aproximación.

Radiación emitida por un haz modulado de partículas cargadas al penetrar en un plasma en un campo magnético uniforme, E. CANOBBIO *(Max-Planck-Institut für Physik und Astrophysik, Munich, República Federal de Alemania)*
Fusión Nuclear 1 (1961) 172—180

La radiación emitida por un haz iónico de densidad modulada al penetrar en un plasma perpendicular a un campo magnético intenso $\mathbf{B}_0$ es estudiada en dos casos simplificados: a) cuando el haz es un plano infinito paralelo a $\mathbf{B}_0$ y b) cuando el haz forma una superficie cilíndrica infinita paralela a $\mathbf{B}_0$, cuyo radio es el radio de giro de las partículas del haz. Este último puede obtenerse idealmente inyectando en un plasma un haz lineal modulado a una frequencia que sea un múltiplo entero de la frecuencia de giro de las partículas del haz y que incida en una dirección que forme un ángulo muy pequeño con un plano perpendicular a $\mathbf{B}_0$.

En ambos casos se observan algunas resonancias del vector de Poynting. El autor estudia especialmente la resonancia que se produce cuando la frecuencia de modulación es igual a la frecuencia de "resonancia iónica", teniendo en cuenta la conductividad eléctrica finita del plasma. Demuestra que en condiciones favorables la interacción haz-plasma se hace muy fuerte para esa resonancia.

Teoría de la radiación de Cerenkov y de la radiación ciclotrónica en los plasmas, T. KIHARA, O. AONO, R. SUGIHARA *(Departamento de Física, Universidad de Tokio, Tokio, Japón)*
Fusión Nuclear **1** (1961) 181—188

Los autores estudian desde el punto de vista teórico, la radiación emitida por una carga q que describe una trayectoria helicoidal en un magnetoplasma. Cuando su velocidad v es mucho mayor que la velocidad térmica $(m^{-1}\,\mathrm{k}T)^{1/2}$ de los electrones del plasma y la frecuencia de rotación mucho menor que la frecuencia ω_0 del plasma, la energía de la radiación procedente de la carga viene dada por la expresión $(q^2\,\omega_0{}^2/2v)\,\ln\,(v^2/m^{-1}\,\mathrm{k}T)$. La radiación ciclotrónica emitida por un electrón de velocidad no relativista decrece hasta cero a medida que la densidad del plasma aumenta. Sin embargo, esa radiación se intensifica para un positrón en un plasma diluido. Esta radiación intensificada del positrón disminuye seguidamente al aumentar el cociente $\omega_0{}^2/\omega_H{}^2$ y se anula para $\omega_0{}^2/\omega_H{}^2 \geq 2$ (ω_H = frecuencia de giro del electrón plasmático). Los autores discuten también la amortiguación de la radiación de Cerenkov debida a las colisiones de electrones plasmáticos.

Radiación ciclotrónica de los iones en un plasma, V. I. PISTUNOVICH, V. D. SHAFRANOV *(Instituto de Energía Atómica "I. V. Kurchatov", Academia de Ciencias de la U.R.S.S., Moscú, U.R.S.S.)*
Fusión Nuclear **1** (1961) 189—194

Los autores determinaron la intensidad de radiación de los iones rápidos en un plasma frío. El problema se planteó con motivo de observaciones, efectuadas en el aparato OGRA, de los máximos de resonancia de la intensidad de un campo eléctrico en regímenes determinados, a saber, a la frecuencia ciclotrónica de los iones y sus armónicos. Aunque las citadas observaciones no se realizaron en la zona ondulatoria, es lógico suponer que la relación de dependencia entre el número de máximos registrados y la densidad del plasma frío, hallada experimentalmente, se reflejaría también en la intensidad de radiación. Los cálculos indicaron que, al aumentar la velocidad de los iones y la densidad del plasma, el máximo de intensidad se desplaza, efectivamente, en el sentido de las frecuencias elevadas.

Este desplazamiento es análogo al desplazamiento del máximo de intensidad de la radiación sincrotrónica de un electrón animado de una velocidad próxima a la de la luz. En el caso de los iones plasmáticos, el papel de la velocidad de la luz lo desempeña la velocidad fásica de las ondas electromagnéticas en el plasma; en la región que reviste importancia para los cálculos, esta velocidad se aproxima a la velocidad de Alfvén: $c_A = B_0/\sqrt{4\pi\,m_i\,n_0}$. Por consiguiente, los armónicos de orden superior se manifiestan ya en correspondencia con una velocidad iónica relativamente baja, a saber, $v \sim c_A \ll c$.

Medición de la temperatura electrónica de un plasma en una onda de choque de gran intensidad. T. I. FILIPPOVA, N. V. FILIPPOV, V. V. YURIN, V. P. VINOGRADOV *(Instituto de Energía Atómica "I. V. Kurchatov" Academia de Ciencias de la U.R.S.S., Moscú, U.R.S.S)*
Fusión Nuclear **1** (1961) 195—197

Aplicando el método de desplazamiento del flujo magnético, los autores han medido la conductividad electrónica de un plasma de deuterio detrás de una fuerte onda de choque. Comprobaron que a velocidades de propagación de $0{,}9 \times 10^7$ y $1{,}25 \times 10^7$ cm/s correspondían temperaturas electrónicas de 50 y 90 eV, respectivamente. Se estableció una comparación entre la temperatura electrónica registrada y la calculada a partir de la velocidad de la onda de choque. Las mediciones piezoeléctricas y la constricción del plasma mediante un campo magnético intenso condujeron a un mismo valor para la presión cinética del gas plasmático.

Pérdida de partículas en una descarga constreñida en un campo magnético axial, R. K. JAGGI *(Instituto de Dinámica de los Flúidos y Matemáticas Aplicadas, Universidad de Maryland, College Park, Maryland, Estados Unidos)*
Fusión Nuclear **1** (1961) 198—200

La pérdida de deuterones entre una corriente constreñida y la pared del recipiente fue calculada por G. P. Thomson [Phil. Mag. **32** (1958) 886]. El autor del presente trabajo extiende los límites de dicho cálculo a fin de tener en cuenta el efecto de un campo magnético axial. Llega a la conclusión de que tal campo es capaz de reducir sensiblemente la pérdida de partículas.

Page 79 Figure 2: delete the factor 10^2 in the ordinates label. Similarly, delete the factor 10 in Figure 3.

Page 134 The approximate expression in Eq. (21) and Eq. (22) are incorrect even to first order, inasmuch as the quantity γ, assumed constant, contains n_+. A useful expression for the upper critical current, where background gas density N_0 can be neglected, comes from Eq. (23):

$$(I_1)_{\mathrm{UC}} = (I_0)_{\mathrm{UC}} \left[1 - \frac{\theta_0\, \sigma_{\mathrm{B}}\, L}{\Gamma\, \sigma_{\mathrm{X}}\, v\, V}\right]^2$$

It has been pointed out by I. N. Golovin (P. R. Bell, private communication) that a sufficiently high initial pumping speed, θ_0, can thus reduce the upper critical current significantly.

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Si vous êtes abonné à la revue ou si elle vous parvient à titre officiel, et si vous désirez un exemplaire d'une ou de plusieurs traductions, adressez votre demande au Rédacteur de «Fusion nucléaire», Agence internationale de l'énergie atomique, Kaerntnerring 11, Vienne I, Autriche. Indiquez votre adresse et le numéro «TT» (voir liste ci-dessous) des traductions qui vous seraient utiles.

Переводы

В скором времени ограниченное число экземпляров переведенных текстов статей (без чертежей и формул), появившихся в журнале „Ядерный синтез", будет предоставлено подписчикам журнала или тем официальным учреждениям, центрам по обмену информацией, правительственным библиотекам и т.д., которые получают бесплатные экземпляры журнала.
Если вы являетесь подписчиком или официальным получателем и желаете получить экземпляр одного или более переведенных текстов, обращайтесь по адресу: «The Editor, Nuclear Fusion, International Atomic Energy Agency, Kaerntnerring 11, Vienna I, Austria». Укажите ваш адрес и номер «TT» переведенных текстов (см. таблицу ниже) которые вы желаете получить.

Traducciones

En breve se dispondrá de un número limitado de ejemplares de los textos traducidos (sin cifras ni ecuaciones) de los artículos aparecidos en «Fusión Nuclear» para las personas suscritas a la revista o para los organismos oficiales, centros de intercambio, bibliotecas públicas, etc., que reciben ejemplares gratuitos.
Si está usted suscrito a la revista, o la recibe a título oficial, y desea recibir un ejemplar de uno o más textos traducidos, pídalos a la dirección siguiente: «Redactor de Fusión Nuclear, Organismo Internacional de Energía Atómica, Kaerntnerring 11, Viena I, Austria». Indique su dirección y el número «TT» de los textos traducidos (véase la lista a continuación) que desee.

				Translated into*
TT-1	*Nuclear Fusion* **1**	(1960)	3—41	R
TT-2	*Nuclear Fusion* **1**	(1960)	42—46	R
TT-3	*Nuclear Fusion* **1**	(1960)	47—53	E
TT-4	*Nuclear Fusion* **1**	(1960)	47—53	R
TT-5	*Nuclear Fusion* **1**	(1960)	54—61	R
TT-6	*Nuclear Fusion* **1**	(1960)	62—63	R
TT-7	*Nuclear Fusion* **1**	(1961)	82—100	E
TT-8	*Nuclear Fusion* **1**	(1961)	121—124	E
TT-9	*Nuclear Fusion* **1**	(1961)	189—194	E
TT-10	*Nuclear Fusion* **1**	(1961)	195—197	E

* E — English, R — Russian

INFORMATION FOR AUTHORS

Inquiries should be addressed to "The Editor, NUCLEAR FUSION, International Atomic Energy Agency, Vienna I, Austria".

Layout of Manuscripts

Authors should set out their manuscripts as follows:

a) *Form of manuscript:* A manuscript may be submitted in English, French, Russian or Spanish. It should be typed in *double* or *triple spacing*, with *wide margins*, and be on good quality paper: *two additional copies* on thin paper should be attached;

b) *Equations* should be numbered consecutively, using *Arabic numerals* in *brackets* the number being placed at the right side of each equation, e.g. "(2)". Where possible, the exact equation will be used in all translations. It is therefore preferable to use only those symbols which are recognized internationally and to avoid abbreviations of words which are meaningful only in a particular language (e.g. E_{eff}, t_{max}, etc.). Simple symbols should be used and their meanings defined in the text.

c) *Figures:*
   1) Each figure should be on a *separate page* and numbered consecutively with *Arabic numerals*, e.g. "Fig. 1" etc.;
   2) Captions for all figures should be listed on a *separate page* and numbered appropriately;
   3) Photographs (glossy prints) should not be sent unless they are indispensable;
   4) Line drawings are preferred, if possible black ink on white tracing paper;
   5) To facilitate translation, internationally-recognized symbols only should be used on all figures, their definitions to be included in the captions.

d) *Footnotes* should be numbered consecutively with *Arabic numerals* and marked within the text where they occur. They should then be listed on a separate page.

e) *References* should be marked in the text consecutively in *Arabic* numerals in square brackets, and the full references listed in numerical order on a *separate page*. Examples of the forms to be used: "[1] SMITH, A. B., *Phys. Rev.* **206** (1955) 483" or "[2] JONES, L. M., Plasma Physics (XYZ Book Co., New York, 1951) 59".

f) *Tables* should be numbered consecutively with *Roman numerals*. The captions should be listed on a *separate page*. Wherever possible internationally-recognized symbols should be used in the column and their meanings defined in the text of the captions. Dimensions should be given in the table captions.

g) *An author's summary* (not more than 300 words) of each article should be included with the manuscripts. The summary will be translated and will appear in the journal in the four official languages of the Agency.

Note:

One original and two copies of all figures, tables and separate pages of footnotes and references are necessary.

When submitting the manuscript, the author should give the name of the person to whom the galley proofs should be sent, will the exact address. Reprint order forms will be sent with the galley proofs.

If the manuscript is accepted, 50 free reprints will be sent to the author. Any additional reprints *must* be ordered when the author returns the galley proofs.

RENSEIGNEMENTS A L'USAGE DES AUTEURS

Les demandes de renseignements doivent être adressées au «Rédacteur en chef, FUSION NUCLEAIRE, Agence internationale de l'énergie atomique, Vienne I (Autriche)»

Présentation des manuscrits

Les auteurs voudront bien se conformer aux dispositions suivantes:

a) *Forme du manuscrit.* Les manuscrits peuvent être rédigés en anglais, français, espagnol ou russe. Ils doivent être dactylographiés *à double* ou *triple interligne*, avec de *grandes marges* et sur du papier de bonne qualité; *deux copies supplémentaires* sur papier pelure seront jointes à chacun d'eux.

b) *Les équations* doivent être numérotées selon l'ordre dans lequel elles sont données, à l'aide de *chiffres arabes* entre *parenthèses*, le chiffre se trouvant à droite de chacune d'elles. Dans la mesure du possible, l'équation sera reproduite exactement dans toutes les traductions. Il est donc préférable de n'utiliser que des symboles reconnus sur le plan international et d'éviter les abréviations qui n'ont de signification que dans une langue déterminée (par exemple: V_{im}, V_{ret}, etc.). Il importe d'utiliser des symboles simples et de préciser leur signification dans le texte.

c) *Croquis.*
   1) Chaque croquis doit figurer sur une *page distincte:* tous les croquis doivent être numérotés, à l'aide de *chiffres arabes*, selon l'ordre dans lequel ils sont présentés, par exemple: Fig. 1, etc.
   2) Les légendes de tous les croquis doivent figurer sur une *page distincte* et être numérotées des manière appropriée.
   3) Des photographies ne doivent être envoyées que si elles sont indispensables.
   4) Il est préférable de soumettre des dessins au trait, si possible à l'encre noire sur papier calque blanc.
   5) Pour faciliter la traduction, il est recommandé de n'utiliser, pour tous les croquis, que des symboles reconnus sur le plan international, et dont la définition devra figurer dans les légendes.

d) *Les renvois à des notes* doivent être indiqués dans le texte, en *chiffres arabes entre crochets*, dans l'ordre numérique. Les notes elles-mêmes figureront sur une page distincte.

e) *Les renvois à des références* doivent être indiqués dans le texte, *en chiffres arabes*, dans l'ordre numérique; les références elles-mêmes figureront dans l'ordre numérique sur une *page distincte*. Exemples des formules à utiliser: «[5] DURAND, P., C. R. Acad. Sci. Paris, **376** (1956) 1344» ou «DUPONT, J., Physique des plasmas, (Nathan, Paris 1957) 59».

f) *Les tableaux* doivent être numérotés en *chiffres romains*, selon l'ordre dans lequel ils sont présentés. Les légendes doivent être reproduites sur une *page distincte*. Dans toute la mesure du possible, il convient d'employer des symboles reconnus sur le plan international et de définir leur signification dans le texte des légendes. Les dimensions doivent être indiquées dans les légendes des tableaux.

g) *Un résumé* (300 mots environ) de chaque article doit être joint au manuscrit. Le résumé sera traduit et publié dans la revue dans les quatre langues de travail de l'Agence.

Note

Il est nécessaire de fournir un original et deux copies de tous les croquis, tableaux et pages de notes et de références.

En soumettant son manuscrit, l'auteur indiquera le nom de la personne à laquelle les premières épreuves devront être envoyées en même temps que son adresse exacte. Un bon de commande pour les tirés à part sera joint aux épreuves. Cinquante tirés à part de chaque article publié seront envoyés gracieusement à l'auteur. S'il désire recevoir des exemplaires supplémentaires, il devra en faire la demande lorsqu'il renverra les épreuves.

ИНФОРМАЦИЯ ДЛЯ АВТОРОВ

За справками следует обращаться по адресу: «Редактору журнала «Ядерный синтез», Международное агентство по атомной энергии, Вена I, Австрия.»

Порядок оформления рукописей

Авторам предлагается следующий порядок оформления рукописей:

a) *Форма рукописи.* Рукопись может быть представлена на английском, русском, французском или испанском языке. Текст рукописи должен быть отпечатан на пишущей машинке *через два или три интервала*, иметь *широкие поля* и быть представлен на бумаге хорошего качества в *трех экземплярах*.

b) *Уравнения* должны быть последовательно пронумерованы *арабскими цифрами в скобках* которые должны быть расположены справа от уравнения, напр. «(2)». По возможности уравнения будут даваться в одинаковой форме в переводах на все языки. Поэтому желательно использовать в уравнениях только международные знаки и избегать также сокращений слов, которые понятны только в данном языке (напр. $E_{пар}$, $X_{макс}$ и т.д.). Следует использовать простые знаки с пояснением в тексте.

c) *Рисунку и схемы.*
   1) Каждый рисунок и схема должны даваться на отдельной странице и пронумерованы последовательно *арабскими цифрами*, напр., «Рис. 1».
   2) Надписи ко всем рисункам и схемам должны быть даны на *отдельном листе* и соответственно пронумерованы.
   3) Направлять фотоснимки следует только в случае крайней необходимости в них.
   4) Желательно представлять чертежи, выполненные черной тушью на белой восковке.
   5) На всех рисунках желательно использовать международные условные знаки. Определения этих знаков должны включаться в подрисуночные надписи. Это облегчит перевод материалов.

d) *Сноски* должны быть пронумерованы *арабскими цифрами* последовательно и указываться в тексте. Затем все сноски должны быть даны на отдельном листе.

e) *Наименования справочной литературы* должны быть пронумерованы *арабскими цифрами* в квадратных скобках. Полные наименования всех использованных изданий должны быть приведены в очередном порядке на отдельной странице по следующему образцу.
«[1] ЧЕХОВ, К., *Атомная энергия* **14** (1959) 137» или «[2] ИВАНОВ, Ф., *Физика плазмы* (АН СССР, 1956)».

f) *Таблицы* должны быть пронумерованы последовательно *римскими цифрами*. Надписи к таблицам должны быть перечислены на отдельном листе. По возможности в таблицах следует использовать международные условные знаки, значения которых должны быть даны в тексте надписей к таблицам. Размеры должны также даваться в тексте надписей к таблицам.

g) К рукописи каждой статьи должна быть приложена краткая аннотация (примерно 300 слов). Аннотация будут переводиться на рабочие языки Агентства и включаться в журнал.

Примечание:

Все рисунки, схемы, таблицы, листы со сносками к тексту и справочной литературной необходимо представлять в трех экземплярах.

При представлении рукописи автор должен указать, кому и по какому адресу должны быть направлены гранки статьи. Бланки на заказ дополнительного количества экземпляров будут направляться одновременно с гранками.

В случае принятия рукописи автору будет бесплатно направляться 50 оттисков. Дополнительные оттиски *следует заказывать* при возвращении в Агентство гранок статьи.

INSTRUCCIONES PARA LOS COLABORADORES

La correspondencia debe dirigirse al «Redactor de la revista FUSION NUCLEAR, Organismo Internacional de Energía Atómica, Viena I, Austria».

Presentación de los Originales

Al preparar los originales deberán tenerse en cuenta las siguientes normas:

a) *Texto de los artículos:* Los artículos podrán redactarse en español, francés, inglés o ruso. El texto, con dos copias, se enviará mecanografiado a *doble* o *triple espacio*, con *amplio margen* y en papel de buena calidad.

b) Todas las *ecuaciones* se numerarán con *números arábigos* colocados entre *paréntesis* a la derecha de cada ecuación. En la medida de lo posible, las ecuaciones se reproducirán exactamente en todas las traducciones; por ello, conviene emplear los símbolos reconocidos internacionalmente y evitar el uso de abreviaturas que sólo tengan sentido en un determinado idioma (por ejemplo, V_{med}, t_{max}, etc.). Deberán utilizarse símbolos simples cuyo significado se definirá en el texto.

c) *Figuras:*
   1) Las figuras se presentarán en *hojas aparte* y se numerarán con *números arábigos:* por ejemplo: «Fig. 1».
   2) Los textos que acompañen a las figuras se agruparán en una *hoja aparte* con la numeración correspondiente.
   3) Sólo se enviarán fotografías cuando sea indispensable.
   4) Se recomienda que las figuras se hagan en dibujo lineal, utilizando en lo posible tinta negra y papel de dibujo blanco.
   5) Para facilitar el trabajo de traducción conviene que en las figuras sólo se empleen símbolos reconocidos internacionalmente; estos símbolos se definirán en el texto que vaya al pie de la figura.

d) Las *notas de pie de página* se indicarán en el texto con *números arábigos* y se agruparán en una hoja aparte.

e) Las *fuentes bibliográficas* se indicarán en el texto con *números arábigos* colocados entre *corchetes*, y se agruparán en una *hoja aparte* en orden numérico. Las menciones se harán de conformidad con los siguientes ejemplos: «[1] GRANADOS, A. J., *Rev. esp. Fís.* **17** (1958) 483» o «[2] JONES, L. M., Plasma Physics (XYZ Book Co., New York, 1957) 59».

f) Los *cuadros* se numerarán con *números romanos* y los textos explicativos se agruparán en una *hoja aparte*. Siempre que sea posible se utilizarán en las columnas símbolos aceptados internacionalmente, dando su significado en los textos explicativos.

g) Junto con el original se enviará un *resumen* (de unas 300 palabras). El resumen se publicará en la revista en los cuatro idiomas de trabajo del Organismo.

Nota:

Deberán enviarse un original y dos copias de las figuras y de los cuadros, así como de las hojas separadas que contengan las notas de pie de página y las fuentes bibliográficas.

Al enviar el original se indicará el nombre y la dirección exacta de la persona a la que deberán remitirse las pruebas de imprenta. Con las pruebas se remitirá un formulario para encargar las separatas.

Los autores de los artículos publicados en la revista recibirán gratuitamente 50 separatas. Si desean recibir una cantidad mayor *habrán de* indicarlo al devolver las pruebas de imprenta.

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